1. Solve the right triangle given that $\alpha = 37^\circ$ and $b = 24$. (find all the missing parts)
   Find the area

2. Solve the right triangle given that $\beta = 16^\circ$ and $a = 31$. (find all the missing parts)
   Find the area

3. Solve the right triangle given that $\alpha = 74^\circ$ and $c = 32$. (find all the missing parts)
   Find the area

Solve the equation. Give all answers between $[0, 2\pi]$ for extra credit.

4. $1 + \sin \theta = 2 \cos^2 \theta$

5. $\sin \left(\frac{3\theta + \pi}{18}\right) = 1$

6. $\sec \left(\frac{3\theta}{2}\right) = -2$

Verify the identity

7. $(\sec x + \tan x)^2 = \frac{1 + \sin x}{1 - \sin x}$

8. $\frac{(\sec^2 \theta - 1) \cot \theta}{\tan \theta \sin \theta + \cos \theta} = \sin \theta$

9. An object attached to a coiled spring is pulled down a distance $a$ from its rest position and then released. Assuming that the motion is simple harmonic with frequency $f$, write an equation that relates the displacement $d$ of the object from its rest position after $t$ seconds. Also assume that the positive direction of the motion is up.
   
   $a = 5$ $f = \frac{1}{6}$ seconds

10. If the distance $d$ (in feet) that an object travels in time $t$ (in seconds) is given by the equation $d = 2 \cos(4t)$, then
    a) Describe the motion of the object.
    b) What is the maximum displacement from its rest position?
    c) What is the time required for one oscillation?
    d) What is the frequency?

11. Two homes are located on opposite sides of a small hill. To measure the distance between them, a surveyor walks a distance of 50 feet from house A to point C, uses a transit to measure angle ACB, which is found to be 90 degrees, and then walks to house B, a distance of 60 feet. How far apart are the houses?

12. Graph one cycle of $\frac{1}{3} \sec\left(x - \pi\right) - 2$. Label the five points (high, low, endpoints, and middle point). Give the amplitude, period, and phase shift.

13. Graph one cycle of $-4 \cot\left(\frac{1}{8} \cdot x + \frac{\pi}{4}\right)$. Label the five points (high, low, endpoints, and middle point). Give the amplitude, period, and phase shift.

14. Use addition or subtraction formulas to find the exact solution. $\sin \frac{19\pi}{12}$

15. If $\sin \alpha = -\frac{4}{5}$ where $\alpha$ is in QIII and $\sec \beta = -\frac{24}{7}$ where $\beta$ is in QII, find:
    a) $\sin(\alpha + \beta)$
    b) $\tan(\alpha - \beta)$
SOLUTIONS:

1. \( \beta = 53^\circ, \quad a = 18.1, \quad c = 30.1 \quad \text{Area} = 217.2 \)

2. \( \alpha = 74^\circ, \quad b = 8.9, \quad c = 32.2 \quad \text{Area} = 137.95 \)

3. \( \beta = 16^\circ, \quad a = 30.8, \quad b = 8.8 \quad \text{Area} = 135.5 \)

4. \( \frac{\pi}{6} + 2\pi k, \quad \frac{5\pi}{6} + 2\pi k, \quad \frac{3\pi}{2} + 2\pi k : \quad EC \quad \frac{\pi}{6}, \quad \frac{5\pi}{6}, \quad \frac{3\pi}{2} \)

5. \( \frac{4\pi}{27} + \frac{2\pi k}{3} : \quad EC \quad \frac{4\pi}{27}, \quad \frac{22\pi}{27}, \quad \frac{40\pi}{27} \)

6. \( \frac{4\pi}{9} + \frac{4\pi k}{3}, \quad \frac{8\pi}{9} + \frac{4\pi k}{3} : \quad EC \quad \frac{4\pi}{9}, \quad \frac{8\pi}{9}, \quad \frac{16\pi}{9} \)

7 & 8 are not given on this worksheet

9. \( d = 5\sin\left(\frac{\pi t}{3}\right) \)

10. a) The point starts 5 units above the origin and moves downward, reaching the origin at \( t = 0.39 \) seconds. It continues moving down with decreasing speed, reaching the low point 5 units below the \( x \)-axis at time \( t = 0.79 \) seconds. The point then reverses direction, moving up and crossing the \( x \)-axis at time \( t = 1.18 \) seconds before reaching to a point 5 units above the \( x \)-axis at time \( t = 1.57 \) seconds

b) 2 feet

c) \( \frac{\pi}{2} \approx 1.57 \) seconds

d) \( f = \frac{2}{\pi} = 0.64 \) oscillations per second

11. 78.1 feet

12. \( \text{amp} = \frac{1}{3}, \quad \text{pd} = 2\pi, \quad \text{ph sh} = \pi, \quad \text{endpts} = \frac{\pi}{2}, \quad \frac{5\pi}{2} \)

13. \( \text{amp} = 4, \quad \text{pd} = 8\pi, \quad \text{ph sh} = -2\pi, \quad \text{endpts} = -2\pi, \quad 6\pi, \quad \text{increasing} \)

14. \( \frac{-\sqrt{2} - \sqrt{6}}{4} \)