MATH 1113
PRACTICE TEST 4

MUST show work on a separate piece of paper.

Name

1. Assume the function is one-to-one. Find the inverse function and check (or prove) your answer. Find the domain and range of \( f \) and \( f^{-1} \).

\[ f(x) = \frac{4x+1}{3x-2} \]

\[ f^{-1} \]

D:____________________ R:____________________

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Evaluate without using a calculator. Show your work!

2. \( \log_{27} \sqrt[3]{9} \)

3. \( \log_{8} \sqrt[3]{32} \)

4. \( \log_{\frac{1}{5}} 3 \)

Solve the following equations. Give the exact solution and estimated solutions.

5. \( 125^{2x+1} = 25^{3x-3} \)

6. \( 2^{2x+3} = 3^{3x+1} \)

7. \( 16(4^x) + 4 = 65(4^x) \)

8. \( 2 = \log_4 (x) + \log_4 (x-6) \)

9. \( \log_{\sqrt{2}} (x+1) + \log_{\sqrt{2}} (2x) = \log_{\sqrt{2}} (x+4) + \log_{\sqrt{2}} (x+1) \)

10. How long does it take $4500 to triple if it is invested at 8.5\% interested compounded quarterly?

11. The size \( P \) of a certain insect population at time \( t \) (in days) obeys the equation \( P = 300e^{0.02t} \). After how many days will the population reach 1500?

12. Radioactive Decay. Iodine-131 is a radioactive material that decays according to the equation \( A = 75e^{-0.087t} \), where time \( t \) is measured in days. What is the half life of Iodine-131?

13. Glottochronology is a method of dating a language at a particular stage, based on the theory that over a long period of time linguistic changes take place at a fairly constant rate. Suppose that a language originally had \( N_v \) basic words and that at time \( t \), measured in millennia (1 millennium = 1000 years), the number \( N(t) \) of basic words that remain in common use is given by \( N(t) = N_v (0.805)^t \). Approximate the percentage of basic words lost every 100 years.
14. One thousand trout, each 1 year old, are introduced into a large pond. It is predicted that the number \( N(t) \) still alive after \( t \) years will be given by the equation \( N(t) = N_0 \left(0.9\right)^t \). When 500 trout will be alive?

15. Radioactive tracer \(^{51}\text{Cr}\) can be used to locate the position of the placenta in a pregnant woman. Often the tracer must be ordered from a medical laboratory. If \( A \) units (microcuries) are shipped, then because of the radioactive decay, the number of units present after \( t \) days is given by the formula: \( A(t) = A e^{-0.0249t} \). If 35 units are shipped and it takes 2 days for the tracer to arrive, approximately how many units will be available for the test?