1. Find all extrema of \( f(x) = x + \sin x \) on the interval \([0, 2\pi]\). (Give exact solutions)

2. Use the first derivative to determine the increasing and decreasing intervals for the function \( f(x) = x\sqrt{4-x^2} \) on the interval \([-2, 2]\). (Give exact solutions)
   
   Critical number(s):
   
   Increasing interval(s):
   
   Decreasing interval(s):

3. Use the graphing utility to graph \( f(x) = 9x^4 + 4x^3 - 36x^2 - 24x \). Use the graph to find the value of the derivative (if it exists) at the extremum between \([0, 2]\).

4. Fill in the blanks and complete the chart, then sketch the graph below.

   \[
   f(x) = \frac{6}{x^2 + 3}
   \]

   Find: Vertical asymptote(s), Horizontal asymptote(s),

   Intercept(s): x-int and y-int, \( f'(x) = \), \( f''(x) = \), Critical number(s),

   Increasing interval(s), Decreasing interval(s), Relative maxima, Relative minima,

   Points of inflection, Concave up, Concave down, Sketch: graph paper provided. If you prefer you can fill in the chart on the paper provided.

5. Decide whether Rolle’s Theorem can be applied to \( f(x) = x^4 - 2x^2 \) on the interval \((-2, 2)\). If Rolle’s Theorem can be applied, find all value(s), \( c \), in the interval such that \( f'(c) = 0 \). If Rolle’s Theorem cannot be applied, state why. You must state Rolle’s Theorem to receive full credit for this problem.

6. Decide whether the Mean Value Theorem can be applied to \( f(x) = 7 - \frac{6}{x} \) on the interval \((1, 6)\). If the Mean Value Theorem can be applied, find all value(s), \( c \), in the interval such that \( f'(c) = \frac{f(b) - f(a)}{b-a} \). If the Mean Value Theorem cannot be applied, state why. You must state the Mean Value Theorem to receive full credit for this problem.

7. Find the following limits. Give exact answers. Show your work.
   
   a) \( \lim_{x \to -\infty} \left( \frac{2x - 5}{\sqrt{3x^2 + 7x - 9}} \right) \)
   
   b) \( \lim_{x \to \infty} \left( \frac{3 - 2x^3 + 6x^4}{8x^4 + 5x^2 + x - 7} \right) \)

8. Find the Second Derivative of the equation \( f(x) = \frac{2x}{9-x^2} \) and determine intervals of concavity.
SOLUTIONS:

1. Min \((0,0)\); Max \((2\pi,2\pi)\)

2. Critical numbers: \(\pm \sqrt{2}\); increasing: \((-\sqrt{2},\sqrt{2})\); decreasing: \((-2,-\sqrt{2}) \cup (\sqrt{2},2)\)

3. Is this a trick question? Isn’t the value at the extremum 0 for the first derivative? Looking at the graph, a minimum occurs at \((1.41, -58.63)\) and \(0 < 1.41 < 2\) so the minimum is within the domain.

4. VA: none, HA: \(y=0\), x-int: none, y-int: 2, \(f'(x) = \frac{-12x}{(x^2+3)^2}\), \(f''(x) = \frac{36x^2 - 36}{(x^2 + 3)^3}\)

   Critical number(s): -1,1 for \(f'\) and -2, 0, 2 for \(f''\); incr: \((-\infty,0)\); decr \((0,\infty)\)

   Relative maxima (0,2), Relative minima none; Points of inflection \((-1,\frac{3}{2}), (1,\frac{3}{2})\),

   Concave up \((-\infty,-1) \cup (1,\infty)\), Concave down \((-1,1)\), Sketch: graph paper provided.

5. \(c = 0, \pm 1\)

6. \(c = \sqrt{6}\)

7. \(a = \frac{-2}{\sqrt{3}}\); \(b = \frac{3}{4}\)

8. \(f''(x) = \frac{4x(x^2+27)}{(9-x^2)^3}\); concave up: \((-\infty,-3) \cup (0,3)\); concave down: \((-3,0) \cup (3,\infty)\)