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FINAL PRACTICE PROBLEMS FOR COLLEGE ALGEBRA
**LECTURE 1-1 ALGEBRAIC EXPRESSIONS**

<table>
<thead>
<tr>
<th>Base and Exponent</th>
<th>Properties of Exponents: Let $m, n$ be real numbers.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x^3 = x \cdot x \cdot x = x \times x \times x$ (3) factors of $x$</td>
<td>1) $a^n \times a^m = a^{n+m}$ (Product) rule</td>
</tr>
<tr>
<td>$x \overset{\text{3 factors}}{\overset{\text{of}}{\text{}}}$ (x) (\overset{\text{base}}{\text{}})</td>
<td>2) $a^n \div a^m = \frac{a^n}{a^m} = a^{n-m}$ (Quotient) rule</td>
</tr>
<tr>
<td>(\overset{\text{3}}{\text{exponent}})</td>
<td>3) $(a^n)^m = a^{n \cdot m}$ (Power) rule</td>
</tr>
<tr>
<td>(\overset{\text{3}}{\text{factors}}) (\overset{\text{of}}{\text{}}) (x)</td>
<td>4) $(ab)^n = a^n b^n$ (Power) of product</td>
</tr>
<tr>
<td>(\overset{\text{3}}{\text{factors}}) (\overset{\text{of}}{\text{}}) (x)</td>
<td>5) $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}, (b \neq 0)$ (Power) of quotient</td>
</tr>
<tr>
<td>(\overset{\text{3}}{\text{factors}}) (\overset{\text{of}}{\text{}}) (x)</td>
<td>6) $a^0 = 1, (a \neq 0)$ (Zero) power rule</td>
</tr>
<tr>
<td>(\overset{\text{3}}{\text{factors}}) (\overset{\text{of}}{\text{}}) (x)</td>
<td>7) $\frac{1}{a^n} = \left(\frac{1}{a}\right)^n = a^{-n}, (a \neq 0)$ (Negative) power</td>
</tr>
</tbody>
</table>

**Example 1:** Simplify the expressions. Assume that all variables are not zero.

(A) $(-3x^2)^2$ \hspace{1cm} (B) $(x^2y^3)^4$

(C) $(-4x^2y^5)(3xy^3)$ \hspace{1cm} (D) $(-6x^5y^2)(-2x^2y^5)$

(E) $\left(-\frac{2x^4}{y}\right)^3$ \hspace{1cm} (F) $\left(-\frac{2}{x^2}\right)^3$

(G) $\frac{28x^3}{7x}$ \hspace{1cm} (H) $\frac{-36x^6y^7}{12x^5y^3}$

(I) $\frac{12x^5y^2}{3xy^5}$ \hspace{1cm} (J) $7x(6x^2)^0$

(A) $9x^4$ \hspace{1cm} (B) $x^8y^{12}$ \hspace{1cm} (C) $-12x^3y^8$ \hspace{1cm} (D) $12x^2y^7$ \hspace{1cm} (E) $-\frac{8x^{12}}{y^5}$ \hspace{1cm} (F) $-\frac{8}{x^5}$ \hspace{1cm} (G) $4x^2$ \hspace{1cm} (H) $-\frac{3}{y^3}$ \hspace{1cm} (I) $\frac{4x^4}{y^7}$ \hspace{1cm} (J) $7x$
**DEFINITION**: A Term is either a single number or a variable, or numbers and variables multiplied together.

\[
\begin{align*}
4x - 9y + 5 \\
\downarrow & \quad \uparrow \\
\text{expressions} & \quad \text{three terms}
\end{align*}
\]

**DEFINITION**: Terms with exactly the same variables that have the same exponents are **like terms**:

- \(9x^2, 6x^2\) : like terms
- \(9x, 6x^2\) : NOT like term

**COMBINING LIKE TERMS**: Add or subtract the coefficients of the like terms.

\[
3x^2 - 5x + 4 - x^2 + 2x = 2x^2 - 3x + 4
\]

**DEFINITION**: The **greatest common factor** is the largest monomial term which will divide evenly into all of the terms in the expression; the greatest common factor of \(30x^3, 45x^2, 50x^5\) is \(5x^2\)

---

**Example 2**: Simplify the algebraic expressions.

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>H</th>
</tr>
</thead>
<tbody>
<tr>
<td>(4(x + 5) + 2(3x - 2))</td>
<td>(5(x + 2y) + 3(-3x - y))</td>
<td>(6(3x - 4) - 3(2x - 9))</td>
<td>(-4(x + 2) + 3(2x - 5))</td>
<td>(3(x - 1) - \frac{1}{2}(4x + 6))</td>
<td>(\frac{1}{3}(3x - 6) - \frac{2}{5}(-10x + 15))</td>
<td>((-2x^2 + 11x) - (x^2 - 8x + 7))</td>
<td>(12 \left(\frac{1}{3}x - \frac{1}{4}\right) - \frac{1}{4}(8x - 12))</td>
</tr>
<tr>
<td>(10x + 16)</td>
<td>(-4x + 7y)</td>
<td>(12x + 3)</td>
<td>(2x - 23)</td>
<td>(x - 6)</td>
<td>(5x - 8)</td>
<td>(-3x^2 + 19x - 7)</td>
<td>(x - 1)</td>
</tr>
</tbody>
</table>
Example 3: Simplify the algebraic expressions:

(A) \(-3x(2x - 5)\) \hspace{1cm} (B) \(x^2(4 - 5x)\)

\(\begin{align*}
(A) & \quad -6x^2 + 15x \\
(B) & \quad -5x^3 + 4x^2 \\
(C) & \quad 4x^4 - 12x^3 \\
(D) & \quad -2x^3 + 10x^2 + 6x \\
(E) & \quad -6x^2y^2 + 8xy^3 \\
(F) & \quad 10x^2y - 30x^2 \\
(G) & \quad 20x^2y - 12xy^2 \\
(H) & \quad 20x^2y^3 - 6x^2y^6
\end{align*}\)

(C) \(4x^3(x - 3)\) \hspace{1cm} (D) \(-2x(x^2 - 5x - 3)\)

(E) \(-2xy^2(3x - 4y)\) \hspace{1cm} (F) \(5x^3(2xy - 6x^2)\)

(G) \(4xy(5x - 3y)\) \hspace{1cm} (H) \(2xy^2(10x^2y^3 - 3xy^4)\)

Example 4: Simplify the algebraic expressions:

(A) \((2x - 5y)(3x + 4y)\) \hspace{1cm} (B) \((5x + 2)(5x - 2)\)

\(\begin{align*}
(A) & \quad 6x^2 - 7xy - 20y^2 \\
(B) & \quad 25x^2 - 4 \\
(C) & \quad 6x^2 + 11x - 35 \\
(D) & \quad -6x^2 + 35xy - 36y^2 \\
(E) & \quad 9x^2 + 30x + 25 \\
(F) & \quad 9x^2 - 24xy + 16y^2
\end{align*}\)

(C) \((2x + 7)(3x - 5)\) \hspace{1cm} (D) \((4y - 3x)(2x - 9y)\)

(E) \((3x + 5)^2\) \hspace{1cm} (F) \((3x - 4y)^2\)
SQUARE ROOT:

\[ x = \sqrt{a} \text{ where } a > 0, \text{ means } \]

a positive real number \( x \) such that \( x^2 = a \)

NOTE: When \( a, b \geq 0 \)

\[ (\sqrt{a})^2 = a, \quad (-\sqrt{a})^2 = a \]

\[ \sqrt{a^2} = |a| = \begin{cases} -a, & a < 0 \\ a, & a \geq 0 \end{cases} \]

PROPERTIES OF SQUARE ROOTS

Let \( a \geq 0, \ b \geq 0 \) and let \( m, n \) be rational numbers.

\[ \sqrt{a} \cdot \sqrt{b} = \sqrt{ab} \]

\[ \frac{\sqrt{a}}{\sqrt{b}} = \frac{\sqrt{a}}{\sqrt{b}} \]

\[ n(m\sqrt{a}) = n \cdot m\sqrt{a} \]

\[ (n\sqrt{a})(m\sqrt{b}) = n \cdot m\sqrt{ab} \]

Example 5: Simplify the radical expressions.

<table>
<thead>
<tr>
<th>(A) ( \sqrt{9} )</th>
<th>(B) ( -\sqrt{64} )</th>
<th>(C) ( \sqrt{-0.04} )</th>
<th>(D) ( \sqrt{144} + 25 )</th>
<th>(E) ( \sqrt{9} + \sqrt{16} )</th>
<th>(F) ( \sqrt{-9} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>-8</td>
<td>-0.2</td>
<td>13</td>
<td>7</td>
<td>Not a real number</td>
</tr>
<tr>
<td>(G) ( \sqrt{0.04} )</td>
<td>(H) ( \sqrt{-125} )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.2</td>
<td>-5</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Example 6: Simplify the radical expressions.

<table>
<thead>
<tr>
<th>(A) ( \sqrt{40} )</th>
<th>(B) ( \sqrt{18} )</th>
<th>(C) ( \sqrt{24} )</th>
<th>(D) ( \sqrt{128} )</th>
<th>(E) ( \sqrt{52} )</th>
<th>(F) ( \sqrt{48} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 2\sqrt{10} )</td>
<td>( 3\sqrt{2} )</td>
<td>( 2\sqrt{6} )</td>
<td>( 8\sqrt{2} )</td>
<td>( 2\sqrt{13} )</td>
<td>( 4\sqrt{3} )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>(G) ( \sqrt{0.04} )</th>
<th>(H) ( \sqrt{-125} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>-5</td>
</tr>
</tbody>
</table>
Example 7: Simplify the radical expressions.

(A) \(8\sqrt{49} - 14\sqrt{100}\)  
(B) \(10\sqrt{32} - 6\sqrt{18}\)  
(C) \(-3\sqrt{18} + 3\sqrt{8} - \sqrt{24}\)  
(D) \(-3\sqrt{2} + 3\sqrt{32} - 3\sqrt{8}\)

Example 8: Simplify the radical expressions.

(A) \(\sqrt[4]{77} + \sqrt[4]{11}\)  
(B) \(\sqrt{3} - \sqrt{15}\)  
(C) \(2\sqrt{35} + 4\sqrt{5}\)  
(D) \(2\sqrt{5} + \sqrt{5}\)  

(C) \(2\sqrt{5}(\sqrt{5} + 2)\)  
(D) \((\sqrt{5} - \sqrt{3})(\sqrt{5} + \sqrt{3})\)
Example 9: Rationalize the denominator

\[
\begin{align*}
\text{(A) } & \frac{1}{\sqrt{6}} & \quad \text{(B) } & \frac{9}{2\sqrt{3}} \\
\text{(C) } & \frac{1}{\sqrt{10} - 2} & \quad \text{(D) } & \frac{1}{3 - \sqrt{7}} \\
\text{(E) } & \frac{2 + \sqrt{3}}{2 - \sqrt{3}} & \quad \text{(F) } & \frac{1}{5 + \sqrt{15}}
\end{align*}
\]
EXERCISE 1

1. Expand the expression and simplify
   (A) \((x - 3)(x + 6)\)  
   (B) \((3x - 4)(2x - 3)\)

   (C) \((x + 3)^2\)  
   (D) \((3x - 4)^2\)

2. Simplify the expression
   (A) \(-3\sqrt{64}\)  
   (B) \(\sqrt{80}\)

   (C) \(\sqrt{32}\)  
   (D) \(\sqrt{50}\)

   (E) \(\sqrt{128}\)  
   (F) \(\sqrt{54}\)

   (G) \(\sqrt[3]{-64}\)  
   (H) \(\sqrt[3]{216}\)

   (I) \(\frac{\sqrt{90}}{\sqrt{10}}\)  
   (J) \(\frac{\sqrt{32}}{\sqrt{25}}\)
3. Rationalize the denominator

(A) \( \frac{5\sqrt{3}}{\sqrt{7}} \)  \hspace{5cm} (B) \( \frac{10}{\sqrt{5}} \)

(C) \( \frac{4}{5 - \sqrt{2}} \)  \hspace{5cm} (D) \( \frac{3}{2 + \sqrt{2}} \)
LECTURE 1-2. COMPLEX NUMBERS AND BASIC FACTOR

COMPLEX NUMBERS: (Real numbers are complex number)

- Imaginary Unit: \( i = \sqrt{-1} \) and \( i^2 = -1 \)
- Standard Form of Complex Numbers: \( a + bi \) where \( a, b \) are real numbers
  \[
  \frac{a}{\text{real part}} + \frac{b}{\text{imaginary part}} \cdot i
  \]
- The conjugate of \( z \) is \( \bar{z} = \overline{\bar{a} + bi} = a - bi \)

OPERATIONS IN COMPLEX NUMBERS: Treat \( i \) like a variable excepts \( i^2 = -1 \)

Example 1: Simplify and write it in the standard form.

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(A) ( i^{21} )</td>
<td>(B) ( i^{-13} )</td>
<td>(A) ( i )</td>
</tr>
<tr>
<td>(B) ( -i )</td>
<td>(C) (-1 )</td>
<td>(D) 1</td>
</tr>
<tr>
<td>(E) 3i</td>
<td>(F) 5i</td>
<td>(G) ( 4\sqrt{2}i )</td>
</tr>
<tr>
<td>(H) ( 2\sqrt{3}i )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(E) ( \sqrt{-9} )</td>
<td>(F) ( \sqrt{-25} )</td>
</tr>
</tbody>
</table>

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(G) ( \sqrt{-32} )</td>
<td>(H) ( \sqrt{-12} )</td>
</tr>
</tbody>
</table>

Example 2: Find the conjugate of the following complex number.

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(A) ( 5 - 4i )</td>
<td>(B) ( 2 + 3i )</td>
<td>(A) ( 5 + 4i )</td>
</tr>
<tr>
<td>(B) ( 2 - 3i )</td>
<td>(C) ( 6i )</td>
<td>(D) 7</td>
</tr>
<tr>
<td>(C) ( 6i )</td>
<td>(D) 7</td>
<td></td>
</tr>
</tbody>
</table>
Example 3: Simplify and write it in the standard form.

(A) \((-2 + 4i) + 2(3 - 6i)\)
(B) \(3(2 - 3i) - 2(5 - 6i)\)
(C) \((2 - 5i)(5 - 3i)\)
(D) \((3 - 7i)^2\)
(E) \((3 - 2i)(3 + 2i)\)
(F) \((5 - 4i)^2\)

(A) \(4 - 8i\)
(B) \(-4 + 3i\)
(C) \(-5 - 31i\)
(D) \(-40 - 42i\)
(E) \(13\)
(F) \(9 - 40i\)

Example 4: Simplify and write it in the standard form.

(A) \(\frac{3}{i}\)
(B) \(\frac{2}{4 - 5i}\)
(C) \(\frac{2 - i}{3 + 2i}\)
(D) \(\frac{2 - 5i}{2 - i}\)

(A) \(-3i\)
(B) \(\frac{8}{41} + \frac{10}{41}i\)
(C) \(\frac{4}{13} - \frac{7}{13}i\)
(D) \(\frac{9}{5} - \frac{8}{5}i\)
DEFINITION: The process of writing an algebraic expression as a product of its factors is called factorization or factoring.

Find the greatest common factor among each terms in the expression

\(x^2 + ax + b\): simple trinomials

\(A^2 - B^2 = (A - B)(A + B)\)

\(A^2 + B^2\): Prime factor

Example 5: Factor out the greatest common factor.

<table>
<thead>
<tr>
<th>(A) (8x - 4)</th>
<th>(B) (-3x^2 - 6x)</th>
<th>(A) (4(2x - 1))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(B) (-3x(x + 2))</td>
<td>(C) (5x(2x + 1))</td>
<td>(D) Prime</td>
</tr>
<tr>
<td>(C) (10x^2 + 5x)</td>
<td>(D) (7x^3 - 5)</td>
<td>(E) (6a^2b^2(2b^3 - 3a))</td>
</tr>
<tr>
<td>(F) (8x^3y^2 - 4x^2y^2 + 12x^2y^3)</td>
<td>(G) ((x + y)(x - y))</td>
<td>(H) (xy(a + b)(x - y))</td>
</tr>
<tr>
<td>(I) (4x^2y^3 - 2x^2y)</td>
<td>(J) (a(x^2 - y) + b(y - x^2))</td>
<td>(I) (2x^2y(2y^2 - 1))</td>
</tr>
<tr>
<td>(J) ((x^2 - y)(a - b))</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Example 6: Factor completely

<table>
<thead>
<tr>
<th>(A) (x^2 - 9)</th>
<th>(B) (x^2 + 4)</th>
<th>(A) ((x - 3)(x + 3))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(B) Prime</td>
<td>(C) ((7x - 3y)(7x + 3y))</td>
<td>(D) ((5x - 3)(5x + 3))</td>
</tr>
<tr>
<td>(C) (49x^2 - 9y^2)</td>
<td>(D) (25x^2 - 9)</td>
<td>(E) ((3x - 13)(3x + 13))</td>
</tr>
<tr>
<td>(F) ((ab - 2)(ab + 2))</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(E) (9x^2 - 169)</td>
<td>(F) (a^2b^2 - 4)</td>
<td></td>
</tr>
</tbody>
</table>
Example 7: Factor completely

<table>
<thead>
<tr>
<th>Expression</th>
<th>Factored Form</th>
</tr>
</thead>
<tbody>
<tr>
<td>(A) $x^2 + 3x + 2$</td>
<td>$(x + 1)(x + 2)$</td>
</tr>
<tr>
<td>(B) $x^2 - 7x + 6$</td>
<td>$(x - 1)(x - 6)$</td>
</tr>
<tr>
<td>(C) $x^2 - 7x + 10$</td>
<td>$(x - 2)(x - 5)$</td>
</tr>
<tr>
<td>(D) $x^2 - 7x + 6$</td>
<td>$(x - 9)(x + 1)$</td>
</tr>
<tr>
<td>(E) $x^2 - 13x + 42$</td>
<td>$(x - 6)(x + 7)$</td>
</tr>
<tr>
<td>(F) $x^2 + 15x + 44$</td>
<td>$(x + 3)(x + 7)$</td>
</tr>
<tr>
<td>(G) $x^2 + 10x + 21$</td>
<td>$(x - 1)(x - 6)$</td>
</tr>
<tr>
<td>(H) $x^2 - x - 12$</td>
<td>$(x - 4)(x + 3)$</td>
</tr>
<tr>
<td>(I) $x^2 - x - 30$</td>
<td>$(x - 6)(x + 5)$</td>
</tr>
<tr>
<td>(J) $x^2 + 5x - 36$</td>
<td>$(x - 6)(x + 5)$</td>
</tr>
<tr>
<td>(K) $x^2 - 12x + 36$</td>
<td>$(x - 6)(x - 6)$</td>
</tr>
<tr>
<td>(L) $x^2 - 14x + 48$</td>
<td>$(x - 6)(x - 8)$</td>
</tr>
<tr>
<td>(M) $x^2 + 2x - 24$</td>
<td>$(x - 6)(x + 4)$</td>
</tr>
<tr>
<td>(N) $x^2 - 4x + 24$</td>
<td>$(x - 6)(x + 4)$</td>
</tr>
<tr>
<td>(O) $x^2 - x - 56$</td>
<td>$(x - 8)(x + 7)$</td>
</tr>
<tr>
<td>(P) $x^2 - 9x - 36$</td>
<td>$(x - 12)(x + 3)$</td>
</tr>
<tr>
<td>(Q) $x^2 - 4x - 12$</td>
<td>$(x - 6)(x + 2)$</td>
</tr>
<tr>
<td>(R) $x^2 - 9x + 14$</td>
<td>$(x - 12)(x + 3)$</td>
</tr>
<tr>
<td>(S) $x^2 + 10x + 16$</td>
<td>$(x - 2)(x - 7)$</td>
</tr>
<tr>
<td>(T) $x^2 - 11x - 26$</td>
<td>$(x - 2)(x - 7)$</td>
</tr>
<tr>
<td>(U) $x^2 + x - 20$</td>
<td>$(x + 5)(x - 4)$</td>
</tr>
<tr>
<td>(V) $x^2 - 15x + 50$</td>
<td>$(x + 5)(x - 4)$</td>
</tr>
</tbody>
</table>
EXERCISE 2

1. Simplify and Write it in the standard form.
   (A) \((5 - 7i) + (4i - 9)\)  \hspace{2cm} (B) \((6 - 3i) - (-2 - 3i)\)

   (C) \((5 - 4i)(5 + 4i)\)  \hspace{2cm} (D) \((2 - 5i)(5 - 3i)\)

   (E) \((5 - 4i)^2\)  \hspace{2cm} (F) \((3 - 7i)^2\)

   (G) \(\frac{3-2i}{2+4i}\)  \hspace{2cm} (H) \(\frac{5-3i}{i}\)

2. Factor out the greatest common factor.
   (A) \(12x^2y^3 - 16x^3y\)  \hspace{2cm} (B) \(2x^5 - 8x^4 + 14x^2\)

   (C) \((a - 3)x - (a - 3)t\)  \hspace{2cm} (D) \(3x^3 - 12x^2\)
3. Factor trinomial
(A) $x^2 - 4x - 12$          (B) $x^2 - 9x + 14$

(C) $x^2 + 10x + 16$          (D) $x^2 - 11x - 26$

(E) $x^2 + x - 20$            (F) $x^2 - 15x + 50$

(G) $x^2 - 16$                (H) $x^2 + 2x + 10$

(I) $x^2 - 4x + 4$            (J) $x^2 + 12x + 36$

(K) $x^2 - 49$                (L) $16x^2 - 9y^2$
**Example 1: Factor the expression**

<table>
<thead>
<tr>
<th>Expression</th>
<th>Factored Form</th>
</tr>
</thead>
<tbody>
<tr>
<td>(A) $ax + bx - ay - by$</td>
<td>$(a + b)(x - y)$</td>
</tr>
<tr>
<td>(B) $10px + 15qx + 8py + 12qy$</td>
<td>$(2p + 3q)(5x + 4y)$</td>
</tr>
<tr>
<td>(C) $12ax - 9ay + 8bx - 6by$</td>
<td>$(3a + 2b)(4x - 3y)$</td>
</tr>
<tr>
<td>(D) $x^3 - 5x^2 - 2x + 10$</td>
<td>$(x - 3)(x - 2)(x + 2)$</td>
</tr>
<tr>
<td>(E) $x^3 + 3x^2 + 2x + 6$</td>
<td>$(x - 2)(x^2 - 5)$</td>
</tr>
<tr>
<td>(F) $x^3 - 2x^2 - 5x + 10$</td>
<td>$(x - 3)(x - 2)(x + 2)$</td>
</tr>
<tr>
<td>(G) $x^3 - 3x^2 - 4x + 12$</td>
<td>$(x - 3)(x - 2)(x + 3)$</td>
</tr>
<tr>
<td>(H) $2x^3 - 3x^2 - 18x + 27$</td>
<td>$(2x - 3)(x - 3)(x + 3)$</td>
</tr>
</tbody>
</table>
Example 2: Factor the expression.

(A) \( 5x^3 - 5x^2 - 30x \)  
(B) \((a + 1)x^2 - 2(a + 1)x - 3(a + 1)\)  
(C) \(2x^2 + 22x + 60\)  
(D) \(x^3 - 7x^2 + 6x\)  
(E) \((x - 2)^2 - (x - 2) - 6\)  
(F) \((x + 3)^2 + 2(x + 3) - 8\)  
(G) \(x^4 - x^2 - 12\)  
(H) \(x^4 - 13x^2 + 36\)
Solve $ax^2 + bx + c = 0$, $a \neq 0$

1) First, check that we can use Square root method: “$(\quad)^0 = \text{constant}$” case

2) Second, if we cannot use square root method, use quadratic formula:

$$ax^2 + bx + c = 0 \Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

3) we can factor

---

Example 3: Find all real or complex solutions of the quadratic equation

<table>
<thead>
<tr>
<th>(A) $x^2 = 16$</th>
<th>(B) $x^2 - 36 = 0$</th>
<th>(C) $x^2 + 9 = 0$</th>
<th>(D) $25x^2 = 16$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x = \pm 4$</td>
<td>$x = \pm 6$</td>
<td>$x = \pm 3i$</td>
<td>$x = \pm \frac{4}{5}$</td>
</tr>
<tr>
<td>(E) $5x^2 - 15x = 0$</td>
<td>(F) $3x^2 = 12x$</td>
<td>(G) $x = -2.4$</td>
<td>(H) $x = -4.3$</td>
</tr>
<tr>
<td>(I) $x^2 - 2x = 8$</td>
<td>(J) $x^2 + x - 12 = 0$</td>
<td>(J) $x = -7.4$</td>
<td></td>
</tr>
<tr>
<td>(I) $x^2 - x - 63$</td>
<td>(J) $x^2 + 4x - 28$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Example 4: Find all real or complex solutions of the quadratic equation

(A) $3x^2 + 5x - 2$
(B) $2x^2 + 3 = 7x$

(A) $x = \frac{1}{3}, -2$
(B) $x = \frac{1}{2}, 3$

(C) $x = \frac{1 \pm \sqrt{10}}{2}$
(D) $x = 3 \pm \sqrt{10}$

(E) $x = \frac{4 \pm \sqrt{10}}{2}$
(F) $x = -\frac{1}{2} \pm \frac{1}{2}i$

(G) $x = 2 \pm i$
(H) $x = 3 \pm 2\sqrt{3}$

(C) $4x^2 - 2x + 3 = 0$
(D) $x^2 - 6x = 1$

(E) $2x^2 + 2x + 1 = 0$
(F) $x^2 - 4x + 5 = 0$

(G) $2x^2 - 8x + 13 = 0$
(H) $x(x - 6) = 3$
EXERCISE 3:

1. Factor
   (A) $ax - 3ay - x + 3$
   (B) $x^3 - 2x^2 - 16x + 32$
   (C) $x^3 + 3x^2 - 4x - 12$
   (D) $3x^3 - 2x^2 - 12x + 8$

2. Solve the following equation.
   (A) $x^2 + 16 = 0$
   (B) $(x - 2)^2 = 12$
   (C) $(x + 5)^2 = 32$
   (D) $5x^2 + 20x = 0$
(E) \( 2x^2 - 6x + 3 = 0 \)  
(F) \( 4x^2 - 6x + 5 = 0 \)

(G) \( 2x^2 + 2x + 1 = 0 \)  
(H) \( 4x^2 - 2x + 3 = 0 \)

(I) \( 2x^2 - 8x + 13 = 0 \)  
(J) \( x(x - 6) = 3 \)

(K) \( 5x^2 + 9x = -4 \)  
(L) \( x(x - 4) + 5 = 0 \)
LECTURE 1-4. MORE SOLVING EQUATIONS.

Example 1: Solve the equation.

<table>
<thead>
<tr>
<th>Equation (A)</th>
<th>Equation (B)</th>
<th>Equation (C)</th>
<th>Equation (D)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$3x(2x + 5) = 4(2x + 5)$</td>
<td>$(x + 3)^2 = 4(x + 3)$</td>
<td>$2x^3 = 8x^2$</td>
<td>$x^3 - x^2 - 6x = 0$</td>
</tr>
<tr>
<td>$(A) x = -\frac{5}{2}, \frac{4}{3}$</td>
<td>$(B) x = 1, -3$</td>
<td>$(C) x = 0, 4$</td>
<td>$(D) x = -2, 0, 3$</td>
</tr>
<tr>
<td></td>
<td>$(E) x = 0, \frac{1}{2}, -4$</td>
<td></td>
<td>$(E) x = 0, \frac{2}{3}, -1$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$(F) x = 0, \frac{2}{3}, -1$</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$(G) x = -3, 2, 3$</td>
<td>$(H) x = -2, 2, \frac{3}{2}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$(H) x = -2, 2, \frac{3}{2}$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Equation (E)</th>
<th>Equation (F)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$3x^3 + x^2 - 2x = 0$</td>
<td>$2x^3 - 9x^2 + 4x = 0$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Equation (G)</th>
<th>Equation (H)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x^3 - 2x^2 - 9x + 18 = 0$</td>
<td>$2x^3 - 3x^2 - 8x + 12 = 0$</td>
</tr>
</tbody>
</table>
Example 2: Solve the equation.

(A) $(x - 2)^2 - (x - 2) - 12 = 0$  
(B) $(x + 1)^2 - 3(x + 1) - 18 = 0$  
(C) $x^4 - 13x^2 + 36 = 0$  
(D) $x^4 - 5x^2 + 4 = 0$

(A) $x = -1, 6$  
(B) $x = -4, 5$  
(C) $x = \pm 2, \pm 3$  
(D) $x = \pm 2, \pm 1$

Example 3: Solve the equation.

(A) $\sqrt{2x - 5} = 3$  
(B) $\sqrt[3]{3x + 4} = 2$  
(C) $3\sqrt{x} = 6$  
(D) $\sqrt{x - 2} = 4$

(A) $x = 16$  
(B) $x = \frac{2}{3}$  
(C) $x = 4$  
(D) $x = 18$
Example 4: Solve the equation.

<table>
<thead>
<tr>
<th>Equation</th>
<th>Solution</th>
</tr>
</thead>
</table>
| (A) $\sqrt{2x + 3} = x$ | (A) $x = 3$  
  (B) $\sqrt{8 - 2x} = x$ | (B) $x = 2$  
  (C) $x = 4$  
  (D) $x = 2, 4$ |
| (E) $\sqrt{x^2 - 3x + 1} = x - 2$ | (E) $x = 3$  
  (F) $\sqrt{x^2 - 5x} = x - 3$ | (F) $x = 9$ |
EXERCISE 4

1. Solve the following equation.
   (A) \( x^3 - 5x^2 + 4x = 0 \)  
   (B) \( 2x^3 - 9x^2 + 4x = 0 \)
   
   (C) \( x^3 + x^2 - 4x - 4 = 0 \)  
   (D) \( 2x^3 - 3x^2 - 18x + 27 = 0 \)

2. Solve the following equation.
   (A) \( \sqrt{3x + 4} = x \)  
   (B) \( \sqrt{7x - 12} = x \)
   
   (C) \( \sqrt{x^2 + 2x + 10} = x + 2 \)  
   (D) \( \sqrt[3]{x - 1} = 2 \)
LECUTRE 1-5. DISTANCE, MIDPOINT, AND CIRCLES

DISTANCE AND MIDPOINT between $P(x_1, y_1)$ and $Q(x_2, y_2)$

- The distance $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
- The midpoint $= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

CIRCLE with center $(h, k)$ and radius $r$.

- Standard form: $(x - h)^2 + (y - k)^2 = r^2$
- General form: $x^2 + y^2 + Ax + By + C = 0$

Example 1: Find the distance and Midpoint between the two points.

(A) $(-5,3)$ and $(3,-4)$
(B) $(-5,-4)$ and $(1,2)$

(A) $\sqrt{113}$; $\left(-\frac{1}{2}, -\frac{1}{2}\right)$
(B) $6\sqrt{2}$; $(-2,-1)$
(C) $5$; $\left(\frac{3}{2}, 5\right)$
(D) $\sqrt{85}$; $\left(\frac{7}{2}, -4\right)$

(C) $(2,3)$ and $(-1,7)$
(D) $(8,-5)$ and $(-1,-3)$

Example 2: Find the center and radius of the circle.

(A) $(x + 3)^2 + (y - 4)^2 = 54$
(B) $(x - 5)^2 + (y + 3)^2 = 8$

(A) $(-3,4)$; $r = 3\sqrt{6}$
(B) $(5,-3)$; $r = 2\sqrt{2}$
(C) $(0,-4)$; $r = 3\sqrt{2}$
(D) $(3,0)$; $r = 4$

(C) $x^2 + (y + 4)^2 = 18$
(D) $(x - 3)^2 + y^2 = 16$
Example 3: Find the center and radius of the circle.

(A) $x^2 + y^2 + 4x - 2y + 1 = 0$

(B) $x^2 + y^2 - 6y = 0$

(A) Center: $(-2, 1)$
Radius: 2

(B) Center: $(0, 3)$
Radius: 3

(C) Center: $(3, -1)$
Radius: $\sqrt{11}$

(D) Center: $(-1, 1)$
Radius: $2\sqrt{2}$

(C) $x^2 - 6x + y^2 + 2y - 1 = 0$

(D) $x^2 + 2x + y^2 - 2y - 6 = 0$

Example 4: Write the standard form of the equation of the circle

(A) center $(-3, 2)$ and radius $= \sqrt{15}$.

(B) center $(3, -5)$ and tangent to y axis.

(A) $(x + 3)^2 + (y - 2)^2 = 15$

(B) $(x - 3)^2 + (y + 5)^2 = 9$

(C) $(x - 1)^2 + (y + 2)^2 = 9$

(D) $(x - 5)^2 + y^2 = 68$

(C) center $(1, -2)$ and containing $(4, -2)$.

(D) center $(5, 0)$ and containing $(-3, 2)$
Example 5: Write the standard form of the equation of the circle

(A) two diameter endpoints at \((1,4)\) and \((-5,-2)\)

\[(x + 2)^2 + (y - 1)^2 = 18\]

(B) two diameter endpoints at \((-2,6)\) and \((6,2)\)

\[(x - 2)^2 + (y - 4)^2 = 20\]

(C) two diameter endpoints are \((1,3)\) and \((4,7)\).

\[(x - \frac{5}{2})^2 + (y - \frac{5}{2})^2 = \frac{25}{4}\]
EXERCISE 5

1. Find the distance and Midpoint between the two points.
   (A) (−5,2) and (3, −4)  (B) (−5, −1) and (1, −3)
   (C) (−2,3) and (2,7)  (D) (3, −1) and (−1,5)

2. Find the center and radius of the following circle.
   (A) \((x - 5)^2 + (y + 3)^2 = 16\)  (B) \((x + 3)^2 + (y - 2)^2 = 24\)
   (C) \(x^2 + 16x + y^2 - 4y + 16 = 0\)  (D) \(x^2 - 2x + y^2 + 8y + 8 = 0\)
   (E) \(x^2 + 10x + y^2 + 13 = 0\)  (F) \(x^2 + y^2 - 6y - 4 = 0\)
3. Find the equation of circle such that

(A) Center: \((3, -4)\), Radius: 4

(B) Center: \((2, -3)\), Contain a point \((-1, 2)\)

(C) Center: \((-2, 3)\), Contain a point \((1, 6)\)

(D) Two diameter end points at \((-4, 3)\) and \((2, -1)\)

(E) Two diameter end points at \((5, 1)\) and \((-1, -3)\)
PRACTICE PROBLEMS FOR UNIT 1

1. Simplify each of the following. Write your final answer in standard \( a + bi \) form.
   - (A) \( \frac{2}{1-2i} \)
   - (B) \( \frac{5-i}{3+2i} \)
   - (C) \( i^{-23} \)
   - (D) \( i^{14} \)

2. Simplify the following. Write your final answer in standard \( a + bi \) form.
   - (A) \( (5 - 3i) - (8 - 9i) \)
   - (B) \( (13 + 2i) + (5 - 8i) \)
   - (C) \( (3 - 7i)(2 - i) \)
   - (D) \( (4 - 7i)^2 \)

3. Simplify each of the following expressions.
   - (A) \( 4x - [2y - 3(3x - 4y)] \)
   - (B) \( -3x^3(4x^2 + 7x - 5) \)
   - (C) \( (3k - 6)(2k + 1) \)
   - (D) \( (5p^2 + 3p)(p^3 - p^2 + 5) \)
   - (E) \( (6m - 5)(6m + 5) \)
   - (F) \( (2r + 5t)^2 \)

4. Factor each of the following expressions.
   - (A) \( p^2 + 4p + pq + 4 \)
   - (B) \( q^2 + 6q - 27 \)
   - (C) \( y^2 - 13y + 40 \)
   - (D) \( x^2 + 3x - 28 \)
   - (E) \( r^2 - r - 56 \)
   - (F) \( x^2 - 2x + 24 \)
   - (G) \( 8p^3 - 24p^2 - 80p \)
   - (H) \( 3x^4 + 30x^3 + 48x^2 \)
   - (I) \( 49y^2 - 25w^2 \)
   - (J) \( x^2 + 100 \)
   - (K) \( 36x^2 - 25 \)
   - (L) \( x^4 - 81 \)

5. Solve the equation
   - (A) \( x^3 - 4x^2 - 2x + 8 = 0 \)
   - (B) \( y^2 = 8y \)
   - (C) \( x^2 = -15 + 8x \)
   - (D) \( t^2 = 12(t - 3) \)
   - (E) \( 81t^2 - 64 = 0 \)
   - (F) \( (x - 3)^2 = 12 \)
   - (G) \( 2x^2 - 2x + 1 = 0 \)
   - (H) \( 4x^2 - 2x + 1 = 0 \)
   - (I) \( x(x - 4) = 5 \)
   - (J) \( x(x - 6) + 25 = 0 \)
   - (K) \( \sqrt{6} - x = x \)
   - (L) \( \sqrt{5x^4 - 4} = x \)

6. Simplify the following expression. Assume all variables are positive.
   - (A) \( \sqrt{128} \)
   - (B) \( \sqrt{28} \)
   - (C) \( \sqrt{108} \)
   - (D) \( \sqrt{45 - \sqrt{125 + \sqrt{500}}} \)
   - (E) \( \sqrt{72} - 3\sqrt{18} + 2\sqrt{8} \)
   - (F) \( 3\sqrt{2} - 2\sqrt{32} + \sqrt{98} \)

7. Rationalize the denominator: \( \frac{5}{4 - \sqrt{10}} \)

8. Find the midpoint and distance between \((-3,2)\) and \((2,-4)\)

9. Find the center and radius of the following circle
   - (A) \( x^2 + y^2 - 4x + 6y + 8 = 0 \)
   - (B) \( x^2 + y^2 + 16x - 2y + 15 = 0 \)

10. Find the equation of a circle such that
    - (A) Center: \((3,-4)\) and contains \((4,2)\)
    - (B) Two diameter end points are \((-4,6)\) and \((2,0)\)
LECTURE 2-1. INTERCEPTS AND SYMMETRY

INTERCEPTS:
- The $x$-intercept is where the graph crosses the $x$ axis; $x$-value when $y$’s value is 0.
- The $y$-intercept is where the graph crosses the $y$ axis; $y$-value when $x$’s value is 0.

Example 1: Identify the $x$-intercept and $y$-intercept of the graph of

(A) $y = \frac{x+3}{x-6}$

(B) $y = \frac{x+8}{x-2}$

(C) $y = \frac{x}{x^2-9}$

(D) $y = x^2 - 4x$

(E) $y = x^2 - 2x - 8$

(F) $y = \sqrt{x} + 4$

<table>
<thead>
<tr>
<th>Equation</th>
<th>$y$-int</th>
<th>$x$-int</th>
</tr>
</thead>
<tbody>
<tr>
<td>(A)</td>
<td>$\frac{1}{2}$</td>
<td>3</td>
</tr>
<tr>
<td>(B)</td>
<td>$-4$</td>
<td>8</td>
</tr>
<tr>
<td>(C)</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>(D)</td>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>(E)</td>
<td>$-8$</td>
<td>4</td>
</tr>
<tr>
<td>(F)</td>
<td>2</td>
<td>$-4$</td>
</tr>
</tbody>
</table>
### SYMMETRY:

<table>
<thead>
<tr>
<th>Point ((a, b))</th>
<th>Symmetry about the (x)-axis</th>
<th>Symmetry about the (y)-axis</th>
<th>Symmetry about the origin</th>
</tr>
</thead>
<tbody>
<tr>
<td>((a, b))</td>
<td>((a, -b))</td>
<td>((-a, b))</td>
<td>((-a, -b))</td>
</tr>
</tbody>
</table>

#### Graph

- Whenever \((a, b)\) in on the graph, \((a, -b)\) is also on the graph.
- Whenever \((a, b)\) in on the graph, \((-a, b)\) is also on the graph.
- Whenever \((a, b)\) in on the graph, \((-a, -b)\) is also on the graph.

#### Algebraic test

- \(f(-x) = f(x)\): even function
- \(f(-x) = -f(x)\): odd function

---

Example 2: Decide whether the following is the symmetry about \(x\)-axis, \(y\)-axis, or origin.

- \((A)\) \(y = x^2 + 2\)
- \((B)\) \(y = x^3 + 2x\)
- \((C)\) \(y = \frac{1}{x}\)
- \((D)\) \(y = \frac{1}{x^2 + 2}\)
- \((E)\) \(y = e^x\)
- \((F)\) \(y = x^3 + 2\)
- \((G)\) \(y = x\)
- \((H)\) \(y = x\), \(y\)-axis, origin

---

![Graph](image-url)
Example 3: Decide whether the following function is odd, even, or neither.

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(A)</td>
<td>$f(x) = x^2 + 2$</td>
<td>(B)</td>
<td>$f(x) = x^3 - 4x$</td>
</tr>
<tr>
<td>(C)</td>
<td>$f(x) =</td>
<td>x + 3</td>
<td>$</td>
</tr>
<tr>
<td>(E)</td>
<td>$f(x) = x^3 + 2$</td>
<td>(F)</td>
<td>$f(x) =</td>
</tr>
<tr>
<td>(G)</td>
<td>$f(x) = \frac{x^2}{x^2+1}$</td>
<td>(H)</td>
<td>$f(x) = 2x^2 - 6x$</td>
</tr>
</tbody>
</table>

Example 4: Show whether the following function is odd, even, or neither, algebraically

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(A)</td>
<td>$f(x) = 3x^2$</td>
<td>(B)</td>
</tr>
</tbody>
</table>

\[ f(-x) = 3(-x)^2 = 3x^2; \text{ even} \]
\[ f(-x) = 2(-x)^3 - 3(-x) = -2x^3 + 3x = -f(x); \text{ odd} \]
\[ f(-x) = \frac{-x}{(-x)^2+3} = \frac{x}{x^2+3} = -f(x); \text{ odd} \]
\[ f(-x) = | -x | + 4 = |x| + 4 = f(x); \text{ even} \]
\[ f(x) = 3(-x^2 - x) = 3x^2 + x; \text{ neither} \]

| (C) | $f(x) = \frac{x}{x^2+3}$ | (D) | $f(x) = |x| + 4$ |

| (E) | $f(x) = 3x^2 - x$ |
EXERCISE 2-1

1. Find the intercepts

   (A) \( y = x^2 - 3x - 4 \)

   (B) \( y = \frac{x-4}{x+8} \)

   (C) \( y = x^4 - 8x^2 - 9 \)

   (D) \( y = \frac{x^2-x-12}{x+5} \)

   (E) \( 2x + 3y = 4 \)

   (F)

2. Decide whether the following is the symmetry about x-axis, y-axis, or origin.

   (A) \( y = |x| + 2 \)

   (B) \( y = x^3 - 4x \)

   (C) \( y = x^2 + 2x \)

   (D) \( y = \frac{x}{x^2+2} \)

   (E)

   (F)

3. Show the \( f(x) = 2x^3 - 5x \) is an odd function, algebraically
LECTURE 2-2 INTRODUCTION TO FUNCTIONS

A relation is any set of ordered pairs.

A function is a correspondence from a first set, called the domain, to a second set, called range, such that each element in the domain corresponds to exactly one element in the range:

- Vertical line test: it is a function if every vertical line intersects the graph in at most one point.
- If we can express this as \( y = \) only one expression of \( x \), then \( y \) is a function of \( x \)

Example 1: Determine whether the following is a function or not

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(A) { (1,5), (2, 5), (3, 8), (4, 7) }</td>
<td>(B) { (4,1), (5, –8), (4, 3), (8, 2) }</td>
</tr>
<tr>
<td>(C) { (-2,5), (3,4), (2,6), (4,5), (1,3) }</td>
<td>(D) { (-1,3), (2,3), (4,3), (5,3) }</td>
</tr>
</tbody>
</table>

Example 2: Determine whether the following is a function or not

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>(A)</td>
<td>(B)</td>
</tr>
<tr>
<td><img src="image1.png" alt="Graph A" /></td>
<td><img src="image2.png" alt="Graph B" /></td>
</tr>
</tbody>
</table>

Example 3: Determine whether the following is a function or not

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(A) ( y = \frac{4-3x}{x+1} )</td>
<td>(B) ( y = x^2 - 3x + 5 )</td>
</tr>
<tr>
<td>(D) ( y = \sqrt{4 - 3x} )</td>
<td>(E) (</td>
</tr>
</tbody>
</table>
### Example 4:
Let \( f(x) = 3x^2 - 7x - 4 \). Find

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>(A) ( f(2) )</td>
<td>(B) ( f(-3) )</td>
<td>(A) ( -6 )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(B) ( 44 )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(C) ( 12x^2 - 14x - 4 )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(D) ( 3a^2 - 19a + 22 )</td>
</tr>
</tbody>
</table>

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<thead>
<tr>
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</thead>
<tbody>
<tr>
<td>(C) ( f(2x) )</td>
<td>(D) ( f(a - 2) )</td>
</tr>
</tbody>
</table>

### Example 5:
Let \( f(x) = 2x^2 - 3x + 5 \). Find

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(A) ( f(2) )</td>
<td>(B) ( f(-3) )</td>
<td>(A) ( 7 )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(B) ( 32 )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(C) ( 50x^2 - 15x + 5 )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(D) ( 2a^2 + 9a + 14 )</td>
</tr>
</tbody>
</table>

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(C) ( f(5x) )</td>
<td>(D) ( f(a + 3) )</td>
</tr>
</tbody>
</table>

### Example 6:
Let \( f(x) = \frac{x^2}{x^2+4} \). Find

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(A) ( f(-2) )</td>
<td>(B) ( f(3x) )</td>
<td>(A) ( 0 )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(B) ( \frac{3x^2}{9x^2+4} )</td>
</tr>
</tbody>
</table>

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Example 7: Find the following value

(A) \( f(3) \) if \( f(x) = \begin{cases} \ x + 3, & x < 2 \\ 2x - 3, & x \geq 2 \end{cases} \)

(B) \( f(-2) \) if \( f(x) = \begin{cases} \ x^2 - 3x, & x < 1 \\ 5 - 2x, & x \geq 1 \end{cases} \)

(C) \( f(4) \) if \( f(x) = \begin{cases} \ x + 2, & x \neq 3 \\ 1, & x = 3 \end{cases} \)

(D) \( f(2) \) if \( f(x) = \begin{cases} \ 3x - 5, & x < -1 \\ 5 - 2x^2, & x \geq -1 \end{cases} \)

(E) \( f(4) \) if \( f(x) = \begin{cases} \ -2x + 4, & x < 4 \\ 3 - x^2, & x \geq 4 \end{cases} \)

Example 8: Draw the graph of the following function;

(A) \( f(x) = \begin{cases} \ 2x - 3, & x < 1 \\ 4 - x, & x \geq 1 \end{cases} \)

(B) \( f(x) = \begin{cases} \ x^3, & x > 1 \\ x + 2, & x \leq 1 \end{cases} \)
EXERCISE 2-2

1. Determine whether the following is a function or not.
   (A) \( y = 3x - 4 \)
   (B) \( y^2 = x \)
   (C) \( \{(2,3), (3,4), (4,-3)\} \)
   (D) \( \{(2,-3), (3,-4), (4,3), (3,2)\} \)

2. Find \( f(3) \) if \( f = \{(-3,2), (-1,3), (1,3), (2,4), (3,2)\} \).

3. Let \( f(x) = 4x - 5 \)
   (A) \( f(4) \)
   (B) \( f(-2) \)
   (C) \( f(x + 3) \)

4. Let \( f(x) = 3x^2 - 7x + 5 \)
   (A) \( f(-3) \)
   (B) \( f(2x) \)
   (C) \( f(x - 3) \)

5. Find the value of the following
   (A) \( f(-2) \) if \( f(x) = \begin{cases} 
   5x - 4, & x < 1 \\
   3 - 2x, & x \geq 1 
\end{cases} \)
   (B) \( f(4) \) if \( f(x) = \begin{cases} 
   x^3 - 2, & x < 2 \\
   3x - 2, & x \geq 2 
\end{cases} \)
   (C) \( f(4) \) if \( f(x) = \begin{cases} 
   3 - 2x, & x < 0 \\
   4, & 0 \leq x < 2 \\
   \frac{3}{2}x^2, & x \geq 2 
\end{cases} \)
HOW TO FIND DOMAIN OF A FUNCTION:

<table>
<thead>
<tr>
<th>Case</th>
<th>Domain</th>
</tr>
</thead>
<tbody>
<tr>
<td>Polynomial case</td>
<td>All real numbers</td>
</tr>
<tr>
<td>Fraction case</td>
<td>All real numbers except the x-values which make denominator zero;</td>
</tr>
<tr>
<td>Square root case</td>
<td>All real numbers which make the inside of square root non-negative;</td>
</tr>
</tbody>
</table>

Example 1: Find the domain of the following function

(A) \( y = 2x - 4x^2 \)  \((A) \ (-\infty, \infty)\)

(B) \( y = \sqrt{12 - 3x} \)  \((B) \ (-\infty, 4]\)

(C) \( y = \frac{x - 2}{x - 4} \)  \((C) \ (-\infty, 4) \cup (4, \infty)\)

(D) \( y = 4x^4 - 5x + 8 \)  \((D) \ (-\infty, \infty)\)

(E) \( y = \sqrt{3x - 15} \)  \((E) \ [5, \infty)\)

(F) \( y = \frac{x - 3}{x + 2} \)  \((F) \ (-\infty, -2) \cup (-2, \infty)\)

(G) \( y = \sqrt{8 - 2x} \)  \((G) \ [4, \infty)\)

(H) \( y = \frac{x + 5}{x^2 - 4} \)  \((H) \ (-\infty, -2) \cup (-2, 2) \cup (2, \infty)\)

(I) \( y = \frac{x + 5}{\sqrt{x - 2}} \)  \((I) \ \)  \((J) \ y = \frac{\sqrt{x + 3}}{x}\)
Example 2: Find the domain and the range of the following relation.

(A) \( D: (-\infty, \infty) \)
(B) \( R: [-2, \infty) \)

Example 3: Let \( f(x) = 2x + 3 \) and \( g(x) = 3x - 4 \). Find

(A) \((f + g)(2)\)
(B) \((f - g)(3)\)
(C) \((f \cdot g)(-2)\)
(D) \(\left(\frac{f}{g}\right)(x)\)
(E) \((f - g)(x)\)
(F) \((f \cdot g)(x)\)

(G) Domain of \((f - g)\)
(H) Domain of \(\frac{f}{g}\)
Example 4: Let \( f(x) = x^2 - x - 2 \) and \( g(x) = x - 4 \). Find

<table>
<thead>
<tr>
<th></th>
<th>((f + g)(-2))</th>
<th>((f - g)(3))</th>
<th>((f - g)(x))</th>
<th>((f \cdot g)(-3))</th>
<th>((\frac{f}{g})(x))</th>
<th>(\text{Domain of } (f - g))</th>
<th>(\text{Domain of } \frac{g}{f})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(A)</td>
<td>-2</td>
<td>3</td>
<td>-70</td>
<td>(x^2-x-2)</td>
<td>(x^2 - 2x + 2)</td>
<td>((-\infty, \infty))</td>
<td>((-\infty, -1) \cup (-1,2) \cup (2,\infty))</td>
</tr>
<tr>
<td>(B)</td>
<td>3</td>
<td>-70</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(C)</td>
<td></td>
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<tr>
<td>(D)</td>
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<tr>
<td>(E)</td>
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<tr>
<td>(F)</td>
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<tr>
<td>(G)</td>
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<td>(H)</td>
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</tr>
</tbody>
</table>
Example 5: Let $f(x) = x^2 - 4$ and $g(x) = \sqrt{x}$. Find

<table>
<thead>
<tr>
<th></th>
<th>(A) $(f + g)(4)$</th>
<th>(B) $(f - g)(1)$</th>
<th>(C) $(f \cdot g)(-2)$</th>
<th>(D) $(\frac{f}{g})(x)$</th>
<th>(E) $(\frac{g}{f})(x)$</th>
<th>(F) Domain of $f - g$</th>
<th>(G) Domain of $\frac{f}{g}$</th>
<th>(H) Domain of $\frac{g}{f}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>14</td>
<td>-4</td>
<td>0</td>
<td>$\frac{x^2 - 4}{\sqrt{x}}$</td>
<td>$\frac{\sqrt{x}}{x^2 - 4}$</td>
<td>$[0, \infty)$</td>
<td>$(0, \infty)$</td>
<td>$[0, 2) \cup (2, \infty)$</td>
</tr>
</tbody>
</table>
EXERCISE 2-3

1. Solve the inequalities and write the answer in the interval form
   (A) $6x - 3 \leq 0$  
   (B) $4 - 5x < 0$

2. Find the domain of the following function.
   (A) $y = 3x - 6$  
   (B) $y = \frac{x}{x+1}$  
   (C) $y = \sqrt{x - 4}$

3. Let $f(x) = 3x - 4$ and $g(x) = 2x - 6$. Find
   (A) $(f + g)(3)$  
   (B) $(f - g)(-1)$  
   (C) $(f \cdot g)(2)$

   (D) $(f + g)(x)$  
   (E) $(f - g)(x)$  
   (F) $\left(\frac{f}{g}\right)(1)$

4. Let $f(x) = x^2 - 2$ and $g(x) = \sqrt{x + 3}$. Find
   (A) $(f + g)(1)$  
   (B) $(f - g)(6)$  
   (C) $(f \cdot g)(-2)$
Lecture 2-4 Properties of Functions and Transformations

Zero(s) of a Function:

- Zeros of a function $f$ are the $x$ value(s) such that $f(x) = 0$
- A point $(a, b)$ is on the graph of $y = f(x)$ if $b = f(a)$

Example 1: Use the graph of a function $y = f(x)$ to answer the following questions

(A) Find $f(-2)$
(B) Find $f(5)$
(C) Find the intercepts of $f$.
(D) Find the domain of $f$.
(E) Find the range of $f$.
(F) For what values of $x$ is $f(x) \geq 0$?
(G) How often does the line $y = 1$ (horizontal line) intersect the graph?
(H) For what values of $x$ is $f(x) = 3$?
(I) Find local maximum/minimum
(J) Find absolute maximum/minimum
(K) Find the increasing interval(s)
Example 2: Write the resulting equation.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(A)</td>
<td>The graph of ( y = \sqrt{x} ) is shifted left by 4 units, reflected across the ( x )-axis, and shifted up by 5 units.</td>
</tr>
<tr>
<td>(B)</td>
<td>The graph of ( y = x^3 ) is shifted left by 3 units, reflected across the ( x )-axis, and shifted up by 3 units</td>
</tr>
<tr>
<td>(C)</td>
<td>The graph of ( y = \sqrt{x} ) is shifted right by 2 units, reflected across the ( x )-axis, and shifted up by 1 unit.</td>
</tr>
<tr>
<td>(D)</td>
<td>The graph of ( y = x^3 ) is shifted right by 5 units, reflected across the ( x )-axis, and shifted down by 7 units.</td>
</tr>
<tr>
<td>(E)</td>
<td>The graph of ( y = x^3 ) is reflected across the ( x )-axis, shifted left by 4 units, and shifted up 5 units.</td>
</tr>
<tr>
<td>(F)</td>
<td>The graph of ( y = x^3 ) is shifted up by 5 units, stretched vertically by factor 4, and shifted left 3 units.</td>
</tr>
</tbody>
</table>

- (A) \( y = -\sqrt{x + 4} + 5 \)
- (B) \( y = -(x + 3)^3 + 3 \)
- (C) \( y = -\sqrt{x - 2} + 1 \)
- (D) \( y = -(x - 5)^3 - 7 \)
- (E) \( y = -(x + 4)^3 + 5 \)
- (F) \( y = 4(x + 3)^3 + 20 \)
Example 3: Decide what kind of transformation we use to get \( g(x) \) from \( f(x) \).

(A) The graph of \( g(x) = 2(x - 4)^2 + 5 \) can be obtained by the transformations of \( f(x) = x^2 \).

(B) The graph of \( g(x) = 3\sqrt{x + 2} - 7 \) can be obtained by the transformations of \( f(x) = \sqrt{x} \).

Example 4: Use the graph of \( f \) to draw each graph.

(A) \( y = f(x) + 2 \)

(B) \( y = f(x - 3) \)

(C) \( y = -f(x) \)

(D) \( y = 3f(x) \)

(E) \( y = f \left( \frac{1}{2} x \right) \)
EXERCISE 2-4

1. Consider \( f(x) = \frac{x-6}{x+3} \).

   (A) Decide whether a point \((-1, -3)\) is on the graph of \( f \)?

   (B) Find \( f(-2) \)

   (C) If \( f(x) = 2 \), what is \( x \)?

   (D) Find \( x \)-intercepts.

   (E) Find the zero(s) of \( f \).

   (F) Find the domain of \( f \)

2. The graph of \( y = x^3 \) is shifted right by 7 units, reflected across the \( x \)-axis, and shifted down by 2 units. Write the resulting equation.

3. The graph of \( y = \sqrt{x} \) is shifted left by 5 units, reflected across the \( x \)-axis, and shifted up by 8 units. Write the resulting equation.

4. The graph of \( y = x^3 \) is shifted down by 3 units, reflected across the \( x \)-axis, and shifted left 4 units. Write the resulting equation.
PRACTICE PROBLEMS FOR UNIT 2

1. Identify the x-intercept and y-intercept of the graph of each function.
   (A) \( f(x) = \frac{x+6}{3-x} \)  \hspace{1cm} (B) \( y = 2x^3 - 7x^2 - 4x \)

2. Find the domain of the following functions
   (A) \( f(x) = 2x^2 - 3x + 4 \)  \hspace{1cm} (B) \( f(x) = \sqrt{x} - 2x \)
   (C) \( f(x) = \frac{x^2}{x-4} \)  \hspace{1cm} (D) \( f(x) = \frac{\sqrt{x}}{x-1} \)
   (E) \( f(x) = \sqrt{5x} - 4 \)  \hspace{1cm} (F) \( f(x) = \frac{x^2}{x^2 + x - 6} \)

3. Decide the symmetry (Draw graph and mention about symmetry)
   (A) \( y = x^2 + 5 \)  \hspace{1cm} (B) \( y = \frac{x}{x^2 + 1} \)
   (C) \( y = x^3 + 3 \)  \hspace{1cm} (D) \( y^2 = x \)

4. Decide whether each is a function or not.
   (A) \( x^2 - y^2 = 4 \)  \hspace{1cm} (B) \( y = \frac{x-2}{x+2} \)
   (C) \( y = x^2 - 3 \)  \hspace{1cm} (D) \( y^2 = x^2 + 3 \)
   (E) \( |y| = x + 2 \)  \hspace{1cm} (F) \( y = \sqrt[3]{x} \)

5. Find the following functions and state the domain of each using \( f(x) = 2x^2 + 1 \) and \( g(x) = 3x - 2 \):
   (A) \( f + g)(x) \)  \hspace{1cm} (B) \( f - g)(4) \)  \hspace{1cm} (C) \( (fg)(x) \)  \hspace{1cm} (D) \( \frac{f}{g}(2) \)

6. Find the domain of \( \frac{f}{g} \) if \( f(x) = \sqrt{x} + 5 \) and \( g(x) = 3 \)

7. Let \( f(x) = 2x^2 - 3x + 5 \). Find the following.
   (A) \( f(-3) \)  \hspace{1cm} (B) \( f(3x) \)  \hspace{1cm} (C) \( f(x - 2) \)  \hspace{1cm} (D) \( -f(x) \)

8. Evaluate \( f(3) \) if \( f(x) = \begin{cases} x^3, & x < 1 \\ \sqrt{x+1}, & x \geq 1 \end{cases} \)

9. Evaluate \( f(-2) \) if \( f(x) = \begin{cases} -2x^2 + 1, & x < -1 \\ 3, & -1 \leq x \leq 1 \\ 2 - 2x, & x > 1 \end{cases} \)
10. For the graph to the right,
   (A) Is it a function?
   (B) If so, find the domain and range.
   (C) \( f(-4) =? \)
   (D) Find the intercepts.
   (E) Find the local maximum 

11. The graph of \( y = x^3 \) is shifted left by 3 units, reflected across the x-axis, and shifted down 9 units. Write the resulting equation.

12. The graph of \( y = \sqrt{x} \) is reflected across the x-axis, shifted right by 4 units, and shifted up 5 units. Write the resulting equation.

13. Find the constant \( k \) if a point \((1, 3)\) is on the graph of \( f(x) = k(x - 3)^3 - 5 \)

14. Consider \( f(x) = \frac{x - 8}{x + 2} \)
   (A) Decide whether a point \((3, -1)\) is on the graph of \( f \)?
   (B) Find \( f(2) \)
   (C) If \( f(x) = 2 \), what is \( x \)?
   (D) Find \( x \)-intercepts.
   (E) Find the zero(s) of \( f \).
   (F) Find the domain of \( f \)
**LECTURE 3-1 LINES**

**LINE:** Let \((x_1, y_1)\) and \((x_2, y_2)\) be two points on a line.

The slope of the line is given by:

\[
\text{Slope of the line} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{f(x_2) - f(x_1)}{x_2 - x_1} = m
\]

**Line equation:**

When slope is defined,

\[
y = m(x - x_1) + y_1 \quad \text{(point-slope form)}
\]

\[
y = mx + b \quad \text{(point-y intercept form)}
\]

When slope is undefined,

\[
x = x_1 \quad \text{(Vertical line)}
\]

<table>
<thead>
<tr>
<th>Vertical line: (x = a)</th>
<th>The other lines: (y = mx + b) where (m) is slope and (b) is its (y) intercept.</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image" alt="Vertical Line" /></td>
<td><img src="image" alt="Other Lines" /></td>
</tr>
</tbody>
</table>

**Slope is undefined**

- \(m > 0\): it is increasing
- \(m = 0\): it is horizontal
- \(m < 0\): it is decreasing

**Example 1:** Find the equation of a line (the linear function) such that

(A) has slope 4 and contains the point \((-1, 3)\)

(B) has slope 2 and contains the point \((3, -1)\)

(C) has slope \(\frac{1}{3}\) and contains the point \((2, -5)\)
Example 2: Find the equation of a line such that

(A) contains the points (−4,3) and (5,−7).

(B) contains the points (2,−1) and (6,−4).

(C) contains the points (4,−3) and (−2,6).

(D) contains the points (−2,−1) and (−1,−6).

(E) has x-intercept 2 and y-intercept 4.

(F) contains the points (2,−1) and (2,4).

(G) contains the points (1,−2) and (2,−2).
**SPECIAL RELATION OF TWO LINES:**

<table>
<thead>
<tr>
<th>Parallel</th>
<th>Perpendicular</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>: Same slopes</strong></td>
<td><strong>: undefined slopes</strong></td>
</tr>
<tr>
<td>$y = m_1x + b_1$</td>
<td>$y = m_2x + b_2$</td>
</tr>
<tr>
<td>$m_1 = m_2$ if $m_1$ and $m_2$ are defined</td>
<td>$x = c_1$, $x = c_2$ if $m_1$ and $m_2$ are undefined</td>
</tr>
<tr>
<td>[x = c_1, x = c_2] if $m_1$ and $m_2$ are undefined</td>
<td></td>
</tr>
</tbody>
</table>

**Example 3:** Find the equation of a line such that

(A) It is parallel to a line $y = 2x - 3$ and contains a point $(-1,2)$.

   (A) $y = 2x + 4$
   (B) $y = -3x + 7$
   (C) $y = -2x + 5$

(B) It is parallel to a line $y = -3x + 5$ and contains a point $(2,1)$.

(C) It is parallel to a line $2x + y = 3$ and it contains a point $(0,5)$.
Example 4: Find the equation of a line such that

<p>| | |</p>
<table>
<thead>
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</thead>
</table>
| (A) It is perpendicular to a line $y = \frac{1}{3}x - 7$ and it contains a point $(2, -3)$ | (A) $y = -3x + 3$
|   |   |
| (B) It is perpendicular to a line $2x - y = 1$ and it contains a point $(1,2)$. | (B) $y = -\frac{1}{2}x + \frac{5}{2}$
|   |   |
| (C) It is perpendicular to a line $4x + 3y = -3$ and it contains a point $(1,3)$. | (C) $y = \frac{3}{4}x + \frac{9}{4}$


EXERCISE 3-1

1. Find the slope and y-intercept of the following line.
   (A) $5x - 7y = 12$    (B) $4x + 3y = 6$

2. Find an equation of a line such that
   (A) slope: $-4$ and contains a point $(2, -3)$

   (B) contains two points $(3, -2)$ and $(-1, 1)$

   (C) contains two points $(-5, -2)$ and $(-5, 1)$

   (D) is perpendicular to a line $y = -2x + 3$ and contains a point $(1, 2)$

   (E) is parallel to a line $2x - y = 4$ and contains a point $(2, 1)$
### Example 1: Solve the system of linear equations.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>(A)</td>
<td>( x + y = 10 ) \ ( x - y = 6 )</td>
</tr>
<tr>
<td>(B)</td>
<td>( 2x + 3y = 4 ) \ ( 4x - 3y = -10 )</td>
</tr>
<tr>
<td>(C)</td>
<td>( 2x - 3y = 5 ) \ ( 3x + y = -9 )</td>
</tr>
<tr>
<td>(D)</td>
<td>( x - 2y = 10 ) \ ( 4x + 7y = 25 )</td>
</tr>
<tr>
<td>(E)</td>
<td>( 5x - 4y = -11 ) \ ( 3x + 2y = 11 )</td>
</tr>
<tr>
<td>(F)</td>
<td>( y = x - 1 ) \ ( 5x - y = 13 )</td>
</tr>
<tr>
<td>(G)</td>
<td>( x = 3y ) \ ( 3x + y = 1 )</td>
</tr>
<tr>
<td>(H)</td>
<td>( 5x + 4y = 21 ) \ ( 4y = x + 15 )</td>
</tr>
</tbody>
</table>

(A)(8, 2) (B)(−1, 2) (C)(−2, −3) (D)(8, −1) (E)(1, 4) (F)(3, 2) (G)(0.3, 0.1) (H)(1, 4)
Example 2: Solve the system of linear equations.

\[
\begin{align*}
(A) \quad x - y &= 8 \\
3x - 3y &= 8 \\
(B) \quad 10x - 4y &= -2 \\
-5x + 2y &= 1 \\
\end{align*}
\]

(A) \(\emptyset\)  
(B) \((x, y) : -5x + 2y = 1\)

Example 3: Solve the application problems

(A) A car rental agency charges a weekly rate of $200 for a car and an additional charge of 5 cents for each mile driven. How many miles can you travel in a week for $500?

\(\text{(A) 6000 miles}\)  
\(\text{(B) 840 minutes}\)

(B) A telephone company charges a monthly rate of $50 for 400 minutes and an additional charge of 5 cents for each minute after 400 minutes. How many minutes did you use in May 2017 if you paid $92 in May 2017?
Example 4: Solve the application problems

(A) At a concert, 3 adult tickets and 2 child tickets cost $167 whereas 2 adult tickets and 1 child ticket cost $103. Find the price of an adult ticket.

(B) A movie theater sells tickets for $9.00 each and seniors for $6.5. One evening the theater sold 600 tickets and took in $4597.5 in revenue. How many of seniors’ ticket were sold?

(C) A bank loaned out $100,000. Part of the money earned 10% per year, and the rest of it earned 6% per year. If the total interest received for one year was $7,000, how much was loaned at 10%?

(D) Flying to Kampala with a tailwind a plane averaged 158 km/h. On the return trip the plane only averaged 112 km/h while flying back into the same wind. Find the speed of the wind and the speed of the plane in still air.
EXERCISE 3-2

Example 1: Solve each system of linear equations by using substitution.

(A) \[ \begin{align*} 6x - y &= 11 \\ -2x - 3y &= -7 \end{align*} \]  
(B) \[ \begin{align*} 2x - 3y &= -1 \\ x - y &= 1 \end{align*} \]

(C) \[ \begin{align*} 3x + y &= 9 \\ 5x - 4y &= -2 \end{align*} \]  
(D) \[ \begin{align*} 4x + 3y &= -3 \\ 2x - y &= 11 \end{align*} \]

(E) \[ \begin{align*} y &= -2 \\ 4x - 3y &= 18 \end{align*} \]  
(F) \[ \begin{align*} y &= 5x - 7 \\ -3x - 2y &= -12 \end{align*} \]
2. A car rental agency charges a weekly rate of $250 for a car and an additional charge of 15 cents for each mile driven. How many miles can you travel in a week for $610?

3. The total price of 4 chips and 2 sodas is $15. The total price of 9 chips and 4 sodas is $32.75. Find the price of a chip.

4. A bank loaned out $50,000. Part of the money earned 10% per year, and the rest of it earned 6% per year. If the total interest received for one year was $4,480, how much was loaned at 10%?
LECTURE 3 QUADRATIC FUNCTIONS

Example 1: (a) graph each quadratic function by determining whether the graph opens up or down and by finding its vertex, axis of symmetry, y-intercept, and x-intercepts, if any. (b) Determine the domain and the range of the function. (c) Find maximum/minimum

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
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<tbody>
<tr>
<td>( f(x) = -x^2 + 2 )</td>
<td>( F(x) = -2(x + 3)^2 - 3 )</td>
<td>( f(x) = 3(x - 4)^2 + 5 )</td>
<td>( F(x) = x^2 + 2x - 3 )</td>
</tr>
<tr>
<td>(A)</td>
<td>(B)</td>
<td>(C)</td>
<td>(D)</td>
</tr>
<tr>
<td>(a)(0.2); ( x = 0; ) (0.2); ( (\sqrt{2},0),(0,-2) )</td>
<td>( -\infty,2 )</td>
<td>Max 2</td>
<td>( -\infty,\infty )</td>
</tr>
<tr>
<td>(b)(-\infty,\infty); ( -\infty,\infty )</td>
<td>(c)Max 2</td>
<td>No xint</td>
<td>(c)Min 5</td>
</tr>
<tr>
<td>(E)</td>
<td>(F)</td>
<td>(G)</td>
<td>(H)</td>
</tr>
<tr>
<td>( f(x) = -4x^2 + 8x - 1 )</td>
<td>( F(x) = 2x^2 + 8x + 3 )</td>
<td>( f(x) = x^2 - 10x + 3 )</td>
<td>( F(x) = -2x^2 + 12x + 5 )</td>
</tr>
<tr>
<td>(A)</td>
<td>(B)</td>
<td>(C)</td>
<td>(D)</td>
</tr>
<tr>
<td>(a)(1,3); ( x = 1; ) (0,-1); ( \left(\frac{-2+\sqrt{10}}{2},0\right) )</td>
<td>( -\infty,\infty; ) ( -\infty,3 )</td>
<td>Max 3</td>
<td>( -\infty,\infty; ) ( -5,\infty )</td>
</tr>
<tr>
<td>(b)(-\infty,\infty; ) ( -\infty,3 )</td>
<td>(c)Max 3</td>
<td>(F)</td>
<td>(c)Min -5</td>
</tr>
<tr>
<td>(G)</td>
<td>(H)</td>
<td>(E)</td>
<td>(F)</td>
</tr>
<tr>
<td>( f(x) = x^2 - 10x + 3 )</td>
<td>( F(x) = -2x^2 + 12x + 5 )</td>
<td>( f(x) = -4x^2 + 8x - 1 )</td>
<td>( F(x) = 2x^2 + 8x + 3 )</td>
</tr>
<tr>
<td>(A)</td>
<td>(B)</td>
<td>(C)</td>
<td>(D)</td>
</tr>
<tr>
<td>(a)(5,-22); ( x = 5 ) ( (0.3); (5 \pm \sqrt{22},0) )</td>
<td>( -\infty,\infty; ) ( -22,\infty )</td>
<td>( -\infty,\infty; ) ( -\infty,\infty )</td>
<td>( -\infty,\infty; ) ( -\infty,\infty )</td>
</tr>
<tr>
<td>(b)(-\infty,\infty; ) ( -22,\infty )</td>
<td>(c)Min -22</td>
<td>(E)</td>
<td>(F)</td>
</tr>
<tr>
<td>(H)</td>
<td>(I)</td>
<td>(G)</td>
<td>(H)</td>
</tr>
<tr>
<td>( f(x) = x^2 - 10x + 3 )</td>
<td>( F(x) = -2x^2 + 12x + 5 )</td>
<td>( f(x) = -4x^2 + 8x - 1 )</td>
<td>( F(x) = 2x^2 + 8x + 3 )</td>
</tr>
<tr>
<td>(A)</td>
<td>(B)</td>
<td>(C)</td>
<td>(D)</td>
</tr>
<tr>
<td>(a)(3,23); ( x = 3 ) ( (0.5); (6 \pm \sqrt{22},0) )</td>
<td>( -\infty,\infty; ) ( -\infty,\infty )</td>
<td>( -\infty,\infty; ) ( -\infty,\infty )</td>
<td>( -\infty,\infty; ) ( -\infty,\infty )</td>
</tr>
</tbody>
</table>
Example 2: Determine whether the following function has a maximum or minimum value and find the value of max/min

(A) \( f(x) = -2(x - 3)^2 + 8 \)  
(B) \( f(x) = 3(x + 4)^2 - 5 \)

(C) \( f(x) = -x^2 + 10x - 4 \)  
(D) \( f(x) = 4x^2 - 8x + 3 \)

(A) Max 8  
(B) Min -5  
(C) Max 21  
(D) Min -1

Example 3: Find the equation of quadratic function for which:

(A) Vertex is \((3, -2)\); contains the point \((1, 6)\)

(B) Vertex is \((4, -3)\); contains the point \((2, -1)\)

(C)
EXERCISE 3-3

1. Using the following quadratic functions, answer the questions. (a) graph each quadratic function by determining whether the graph opens up or down and by finding its vertex, axis of symmetry, y-intercept, and x-intercepts, if any. (b) Determine the domain and the range of the function. (c) Find maximum/minimum

(A) \( y = 2(x + 3)^2 + 4 \)

(B) \( y = -3(x - 5)^2 + 2 \)

(C) \( y = 2x^2 - 10x + 9 \)

(D) \( y = -3x^2 + 9x + 2 \)

2. Determine the quadratic function whose vertex is \((2, -3)\) and passes through a point \((-1, 2)\).
Example 1: Solve the application problems

(A) Mary wants to make a rectangular flower bed. If she uses 200 yards of fencing material, find the length and width of the largest flower bed.

(B) Tom has 120 feet of fencing available to enclose a rectangular field. One side of the field lies along the highway, so only three sides require fencing. Find the largest area.

Example 2: A ball is propelled vertically upward with an initial velocity of 20 meters per second. The distance \( h \) (in meters) of the object from the ground after \( t \) seconds is 

\[ h(t) = -5t^2 + 20t. \]

(A) When will the ball be 15 meters above the ground?

(B) When will it strike the ground again?

(C) Will the ball reach a height of 60 meters?
Example 3: A person standing close to the edge on the top of a 512-foot building throws a ball vertically upward. The quadratic function

\[ s(t) = -16t^2 + 64t + 512 \]

models the ball’s height above the ground, \( s(t) \), in feet, \( t \) seconds after it was thrown.

(A) After how many seconds does the ball reach its maximum height?

(B) What is the maximum height?

(C) How many seconds does it take until the ball finally hits the ground?

Example 4: Solve the application problems

(A) The length of a rectangle is 3 cm greater than its width. The area of the rectangle is 108 square centimeters. Find the length of the rectangle.

(B) The perimeter of a rectangle is 24 cm. Find the maximum area of this rectangle.
EXERCISE 3-4

1. A ball is thrown from the top of a building. Its height from the ground, \( h \) meters, after time \( t \) seconds, is given by \( h = 160 + 20t - 5t^2 \).
   
   (A) When does the ball hit the ground?
   (B) When does the ball reach the maximum height?
   (C) What is the maximum height?

2. David has available 500 yards of fencing and wishes to enclose a rectangular area. Find the dimension of the rectangular lot if the area is to be maximum?

3. Tom wants to construct a rectangular flower bed on land bordered on one side by his house. It has 40 ft. of fencing that is to be used to fence off the other three sides. What should be the dimension of the flower bed if the enclosed area is to be maximum? What is the maximum area?
PRACTICE PROBLEMS FOR UNIT 3

1. Solve the following system of linear equations.
   (A) $-2x + 3y = 7$
   $x + 5y = 3$
   (B) $3x + 5y = 1$
   $4x + 7y = 2$
   (C) $y = x + 8$
   $6x - y = 7$
   (D) $2x - y = 11$
   $3x - 4y = 29$
   (E) $\frac{x - y}{2} = 1$
   $6x - 9y = 18$
   (F) $3x - 4y = 12$
   $6x - 8y = 20$

2. The total cost of 4 cutters and 2 glue sticks is $13. The total cost of 7 cutters and 5 glue sticks is $25. Find the price of a cutter and the price of a glue stick.

3. A movie theater sells tickets for $7.00 each and children for $4. One evening the theater sold 140 tickets and took in $731 in revenue. How many of children’ ticket was sold?

4. A bank loaned out $50,000. Part of the money earned 8% per year, and the rest of it earned 5% per year. If the total interest received for one year was $2,950, how much was loaned at 8%?

5. (a) graph each quadratic function by determining whether the graph opens up or down and by finding its vertex, axis of symmetry, y-intercept, and x-intercepts, if any. (b) Determine the domain and the range of the function. (c) Find maximum/minimum
   (A) $f(x) = -2(x + 4)^2 + 5$
   (B) $f(x) = \frac{1}{2}(x - 3)^2 - 2$
   (C) $f(x) = -x^2 - 4x + 6$
   (D) $f(x) = 2x^2 - 8x + 1$
   (E) $f(x) = 2x^2 - 6x + 3$
   (F) $f(x) = -4x^2 - 8x - 5$

6. Find the quadratic equation which satisfies the following conditions
   (A) Its vertex is (2, −3) and it passes through a point (1, 4)
   (B) Its vertex is (−4, 5) and it passes through a point (−2, 3)

7. A ball is thrown upward. The height $H$ of a ball in feet after $t$ seconds in $H(t) = -16t^2 + 192t$.
   (A) When does the ball reach the maximal height?
   (B) Find the maximal height.
   (C) When is the ball 208 ft. above the ground?
   (D) How long does it take the ball to hit the ground?

8. Suppose that the ABAC company discovered that when the unit price is $x$ dollars, the revenue $R$ in dollars is $R(x) = -3x^2 + 3300x$
   (A) What unit price should be established to maximize the revenue?
   (B) What is the maximum of revenue?
9. ABAC decide to make a rectangular flower bed in front of Science building by using 640 yards of fencing available. Find the length and width of the flower bed when it has the largest area.

10. ABAC decide to make a rectangular parking lot by using 860 yards of fencing available. One side of the parking lot lies along the highway, so only three sides require fencing. Find the largest area.

11. The length of a rectangle is 6 cm greater than its width. The area of the rectangle is 27 square centimeters. Find the length of the rectangle.

SOLUTION

1. (A) (−2,1)  (B)(−3,2)  (C)(3,11)  
   (D)(3,−5)  (E){x|6x − 9y = 18}  (F) No solution

2. The price of cutter is $2.50 and the price of glue is $1.50

3. 83

4. $15,000

5. (A) (−4,5);  x = −4;  (0, −27);  \((\frac{9}{2}, 0), (\frac{9}{2}, 0]\)
   (b)(−∞, ∞);  (−5, 0)  (c) Max 5

   (B) (3, −2);  x = 3;  \((0, \frac{5}{2}), (5, 0), (1,0)\)
   (b)(−∞, ∞);  [−2, ∞)  (c) Min − 2

   (C) (a)(−2,10);  x = −2; (0,6); (−2 + √10, 0), (−2 − √10, 0)
   (b)(−∞, ∞);  (−10, 10)  (c) Max 10

   (D) (a)(2,−7);  x = 2;  (0,1);  \((\frac{4+\sqrt{10}}{2}, 0), (\frac{4-\sqrt{10}}{2}, 0)\)
   (b)(−∞, ∞);  [−7, ∞)  (c) Min − 7

   (E) (a) \((\frac{3}{2}, \frac{3}{2})\);  x = \(\frac{3}{2}\);  (0,3);  \((\frac{3+\sqrt{7}}{2}, 0), (\frac{3-\sqrt{7}}{2}, 0)\)
   (b)(−∞, ∞);  \[-\frac{3}{2}, \frac{3}{2}\)  (c) Min − \(\frac{3}{2}\)

   (F) (a)(−1,−1);  x = −1;  (0,−5); No x − int
   (b)(−∞, ∞);  (−1, −1)  (c) Max − 1

6. (A)y = 7(x − 2)^2 − 3  (B)y = −\(\frac{1}{2}\)(x + 4)^2 + 5

7. (A) 6 sec  (B) 576 ft.  (C) 3sec and 9 sec  (D) 12 sec

8. (A) $550  (B) $907,500

9. 160 yd. by 160 yd.

10. 92,450 square yards

11. 9 cm
Lecture 4-1 Polynomials

Example 1: Find the leading coefficient of the polynomial.

<table>
<thead>
<tr>
<th>Option</th>
<th>Polynomial</th>
</tr>
</thead>
<tbody>
<tr>
<td>(A)</td>
<td>( f(x) = x^4 + 2x^2 + x - 7 )</td>
</tr>
<tr>
<td>(B)</td>
<td>( f(x) = -4x^7 + x^6 )</td>
</tr>
<tr>
<td>(C)</td>
<td>( f(x) = 3x^2(x - 3)(x + 2)^2 )</td>
</tr>
<tr>
<td>(D)</td>
<td>( f(x) = -2(x - 2)^3(2x - 1)^2 )</td>
</tr>
</tbody>
</table>

Example 2: Find the zeros and its multiplicity of the functions. Decide whether it crosses or touches the \( x \)-axis at each zero.

<table>
<thead>
<tr>
<th>Option</th>
<th>Polynomial</th>
</tr>
</thead>
<tbody>
<tr>
<td>(A)</td>
<td>( f(x) = 6(x - 4)(x + 5)^2 )</td>
</tr>
<tr>
<td>(B)</td>
<td>( f(x) = -5x^2(x - 1)^2(x + 2)^3 )</td>
</tr>
<tr>
<td>(C)</td>
<td>( f(x) = 3x(x - 5)^6 )</td>
</tr>
<tr>
<td>(D)</td>
<td>( f(x) = 3 \left(x - \frac{1}{2}\right)^2(x + 7)^8 )</td>
</tr>
</tbody>
</table>

Example 3: Find a polynomial whose zeros and degree are given:

<table>
<thead>
<tr>
<th>Option</th>
<th>Zeros</th>
<th>Degree</th>
</tr>
</thead>
<tbody>
<tr>
<td>(A)</td>
<td>(-1) of multiplicity 1, 2 of multiplicity 2, and 3 of multiplicity 2; degree = 5</td>
<td>( (x + 1)(x - 2)^2(x - 3)^2 )</td>
</tr>
<tr>
<td>(B)</td>
<td>2 of multiplicity 2, (-2) of multiplicity 3, and 4 of multiplicity 2; degree = 7</td>
<td>( (x - 2)^2(x + 2)^3(x - 4)^2 )</td>
</tr>
<tr>
<td>(C)</td>
<td>(-1) of multiplicity 2, 2 of multiplicity 1, and (-5) of multiplicity 3; degree = 6</td>
<td>( (x + 1)^2(x - 2)(x + 5)^3 )</td>
</tr>
</tbody>
</table>

Example 4: Which of following is a factor of a polynomial \( 2x^3 - 5x^2 - 4x + 3 \)?

<table>
<thead>
<tr>
<th>Option</th>
<th>Polynomial</th>
</tr>
</thead>
<tbody>
<tr>
<td>(A)</td>
<td>( x + 3 )</td>
</tr>
<tr>
<td>(B)</td>
<td>( x - 1 )</td>
</tr>
<tr>
<td>(C)</td>
<td>( 2x - 1 )</td>
</tr>
<tr>
<td>(D)</td>
<td>( 2x + 1 )</td>
</tr>
</tbody>
</table>

\[ \text{C} \]
Example 5: Which of following is a factor of a polynomial $x^4 - 4x^3 + 3x^2$?

- (A) $x + 3$
- (B) $x + 1$
- (C) $x + 2$
- (D) $x$

Example 6: Find quotient and remainder

- (A) Divide $4x^3 - 7x^2 - 11x + 5$ by $x - 1$.
  - $Q: 4x^2 - 3x - 14, R: -9$
- (B) $Q: x^2 + 3x + 9, R: 0$
- (C) $Q: 3x^2 - 11x - 7, R: -39$
- (D) $Q: 2x^3 - 5x^2 + 5x - 2, R: 3$
- (E) $Q: x^2 - 3x + 5, R: 5$

- (B) Divide $x^3 - 27$ by $x - 3$.

- (C) Divide $3x^3 - 17x^2 + 15x - 25$ by $x - 2$

- (D) Divide $2x^4 - 3x^3 + 3x + 1$ by $x + 1$

- (E) Divide $2x^3 - 9x^2 + 19x - 10$ by $2x - 3$
EXERCISE 4-1

1. Find the zeros and its multiplicity of the functions. Decide whether it crosses or touches the $x$-axis at each zero.
   
   (A) $f(x) = -3x^2(x - 5)^5$

   (B) $f(x) = (2x + 1)^3(x - 3)^4$

2. Find a polynomial whose zeros and degree are given:
   
   (A) Zeros: $-3$ of multiplicity 2, 4 of multiplicity 2, and $-1$ of multiplicity 3; degree = 7

   (B) Zeros: $-1$ of multiplicity 2, 3 of multiplicity 2, and 4 of multiplicity 1; degree = 5

3. Find quotient and remainder
   
   (A) Divide $2x^3 + 5x^2 - 8x + 3$ by $x - 2$.

   (B) Divide $-5x^4 + 2x^2 + 6x - 8$ by $x + 1$. 
**LECTURE 4-2 ZEROS OF POLYNOMIALS**

Example 1: List all possible rational zeros of

<p>| | |</p>
<table>
<thead>
<tr>
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</thead>
<tbody>
<tr>
<td>(A) $f(x) = 2x^3 + 3x^2 - 8x + 5$</td>
<td>(B) $f(x) = x^3 - 9x^2 + 12x - 15$</td>
</tr>
<tr>
<td>(A) $\pm 1, \pm 5, \pm \frac{1}{2}, \pm \frac{5}{2}$</td>
<td>(B) $\pm 1, \pm 3, \pm 5, \pm 15$</td>
</tr>
<tr>
<td>(C) $\pm 1, \pm 2, \pm 3, \pm 5, \pm 10, \pm 15, \pm 30$</td>
<td>(D) $\pm 1, \pm 2, \pm 7, \pm 14$</td>
</tr>
</tbody>
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<tr>
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</tr>
</thead>
<tbody>
<tr>
<td>(C) $f(x) = x^4 - x^3 + 14x^2 - 3x - 30$</td>
<td>(D) $f(x) = x^4 - 3x^2 + 7x + 14$</td>
</tr>
</tbody>
</table>

Example 2: Find the real zeros of the polynomial and write the zeros to factor completely

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<tbody>
<tr>
<td>(A) $f(x) = 2x^3 - 5x^2 - 4x + 3$</td>
<td>(B) $f(x) = 3x^3 + x^2 - 12x - 4$</td>
</tr>
<tr>
<td>(A) $\frac{1}{2}, 3, -1$; $f(x) = (2x - 1)(x - 3)(x + 1)$</td>
<td>(B) $-2, -\frac{1}{3}, 2$; $f(x) = (x + 2)(3x + 1)(x - 2)$</td>
</tr>
<tr>
<td>(C) $\frac{3}{2}$; $f(x) = (2x - 3)(x^2 + 3)$</td>
<td>(D) $2, \frac{3}{2}, \frac{1}{2}$; $f(x) = (x - 2)(2x - 3)(3x - 1)$</td>
</tr>
</tbody>
</table>

Example 3: Solve the equation;

<p>| | |</p>
<table>
<thead>
<tr>
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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(A) $6x^3 + 23x^2 + 11x + 12 = 0$</td>
<td>(B) $15x^3 - 23x^2 - 18x + 8 = 0$</td>
</tr>
<tr>
<td>(A) $\frac{1}{2}, -\frac{1}{3}, -3$</td>
<td>(B) $\frac{1}{2}, -\frac{4}{3}$</td>
</tr>
</tbody>
</table>
Example 4: Find all real or complex zeros of the polynomial

<table>
<thead>
<tr>
<th>Polynomial</th>
<th>Zeroes</th>
</tr>
</thead>
<tbody>
<tr>
<td>(A) $f(x) = x^3 - 2x^2 - 5x + 6$</td>
<td>(A) $-2, 1, 3$</td>
</tr>
<tr>
<td>(B) $f(x) = 2x^3 - 5x^2 - 4x + 3$</td>
<td>(B) $\frac{1}{2}, -1, 3$</td>
</tr>
<tr>
<td>(C) $f(x) = x^4 + x^3 - 12x^2 - 10x + 20$</td>
<td>(C) $1, -2, \pm \sqrt{10}$</td>
</tr>
<tr>
<td>(D) $f(x) = x^3 + 3x^2 - 2x - 6$</td>
<td>(D) $-3, \pm \sqrt{2}$</td>
</tr>
<tr>
<td>(E) $f(x) = x^4 - 3x^3 + 7x^2 + 21x - 26$</td>
<td>(E) $-2, 1, 2 \pm 3i$</td>
</tr>
<tr>
<td>(F) $f(x) = x^4 + 6x^3 + 10x^2 + 6x + 9$</td>
<td>(F) $-3, \pm i$</td>
</tr>
</tbody>
</table>
Example 4: Find a polynomial whose coefficients are real, and zeros and degree are given:

(A) Zeros: 3 of multiplicity 2, $1 - i$ of multiplicity 1; degree = 4

\[(A) \quad (x - 3)^2(x^2 - 2x + 2)\]

(B) Zeros: 2 of multiplicity 1, $3i$ of multiplicity 1; degree = 3

\[(B) \quad (x - 2)(x^2 + 9)\]

(C) Zeros: $2 - i$ of multiplicity 1, $2i$ of multiplicity 1; degree = 4

\[(C) \quad (x^2 - 4x + 5)(x^2 + 4)\]

Example 5: Find the remainder when $f(x)$ is divided by $r(x)$;

(A) $f(x) = 3x^5 + 6x^2 - 7x + 9$; $r(x) = x + 1$

\[(A) \quad 9\]

(B) $f(x) = \frac{1}{4}x^6 - 5x^3 + 7x + 5$; $r(x) = x + 2$

Example 6: Find the value of the constant $k$ if

(A) $f(x) = 3x^3 + 2kx^2 - 4x + 7k$ has a factor $x - 1$.

\[(A) \quad k = \frac{1}{6}\]

(B) $f(x) = 2x^3 - 3kx^2 + 4kx + 5$ has a factor $x + 2$.

\[(B) \quad k = -2\]
EXERCISE 4-2

1. Find the real/complex zeros of the polynomial
   (A) \( f(x) = 2x^4 + x^3 - 14x^2 + 5x + 6 \)
   
   (B) \( f(x) = x^4 + x^3 - 14x^2 - 2x + 24 \)
   
   (C) \( f(x) = x^4 - 3x^3 + 18x^2 - 48x + 32 \)
   
   (D) \( f(x) = x^4 + 8x^3 + 13x^2 - 32x - 68 \)

2. Find a polynomial whose zeros and degree are given:
   (A) Zeros: \(-4\) of multiplicity 1, \(2 - i\) of multiplicity 1; degree = 3
   
   (B) Zeros: 2 of multiplicity 2, \(2i\) of multiplicity 1; degree = 4
LECTURE 4.3 RATIONAL FUNCTIONS

**DOMAIN OF RATIONAL FUNCTION:** If $P(x)$ and $Q(x)$ are polynomials, then a function of the form $f(x) = \frac{P(x)}{Q(x)}$ is called a rational function, provided that $Q(x)$ is not the zero polynomial.

- Zero(s) of $(x)$: all $x$-values $c$ such that $P(c) = 0$ and $Q(c) \neq 0$.
- Domain of $(x)$: all real number except $x$-values $c$ such that $Q(c) = 0$.

**ASYMPTOTE**

(A) Vertical Asymptote; The line $x = a$ is a vertical asymptote for the graph of a function if $f(x) \to \infty$ or $f(x) \to -\infty$ as $x$ approaches $a$ from either the left or the right.

(B) Horizontal Asymptote: The line $y = c$ is a horizontal asymptote for the graph of function $f$ if $f(x) \to c$ as $x \to \infty$ or $x \to -\infty$.

**HOW TO FIND VERTICAL/HORIZONTAL/OBLIQUE ASYMPTOTES?**

Let $f(x) = \frac{P(x)}{Q(x)}$ be a rational function.

(A) Vertical Asymptote; line $x = c$ where $P(c) \neq 0$ and $Q(c) = 0$.

(B) Horizontal Asymptote

(A) If $\deg P(x) > \deg Q(x)$, there are no Horizontal Asymptote.

(B) If $\deg P(x) = \deg Q(x)$, Horizontal Asymptote is

$$y = \frac{\text{Leading coefficient } P(x)}{\text{Leading coefficient } Q(x)}$$

(C) If $\deg P(x) < \deg Q(x)$, line $y = 0$ is Horizontal Asymptote.

- Oblique Asymptote; only when $\deg P(x) = \deg Q(x)+1$,
  oblique asymptote is “$y = \text{quotient}$” when $P(x)$ is divided by $P(x)$.
Example 1: Using the given rational function,

(A) Find the domain.

(B) Find hole if any

(C) Find intercepts.

(D) What is the equation of the vertical asymptote(s) of this function?

(E) What is the equation of the horizontal asymptote(s) of this function?

(F) What is the equation of the oblique asymptote(s) of this function?

1. \( h(x) = \frac{x+2}{x^2+2x-8} \)

1. (A) \((-\infty, -4) \cup (-4,2) \cup (2, \infty)\)

(B) None

(C) \((-2,0), (0, \frac{1}{4})\)

(D) \(x = 2, x = -4\)

(E) \(y = 0\)

(F) None

2. \( g(x) = \frac{x^2-2x-8}{x^2+x-2} \)

2. (A) \((-\infty, -2) \cup (-2,1) \cup (1, \infty)\)

(B) At \(x = -1\)

(C) \((4,0), (0,4)\)

(D) \(x = 1\)

(E) \(y = 1\)

(F) None

3. \( f(x) = \frac{x^2+x-6}{x+2} \)

3. (A) \((-\infty, -2) \cup (-2, \infty)\)

(B) None

(C) \((2,0), (-3,0)\)

(D) \(x = -2\)

(E) None

(F) \(y = x - 1\)
Example 2: Using the given rational function,

(A) Find the domain.
(B) Find hole if any
(C) Find intercepts.
(D) What is the equation of the vertical asymptote(s) of this function?
(E) What is the equation of the horizontal asymptote(s) of this function?
(F) What is the equation of the oblique asymptote(s) of this function?

1. \( g(x) = \frac{3x + 12}{x - 3} \)

   (A) \((-\infty, 3) \cup (3, \infty)\)
   (B) None
   (C) \((-4, 0), (0, -4)\)
   (D) \(x = 3\)
   (E) \(y = 3\)
   (F) None

2. \( h(x) = \frac{x^2 - 2x - 8}{x + 2} \)

   (A) \((-\infty, -2) \cup (-2, \infty)\)
   (B) At \(x = -2\)
   (C) \((4, 0), (0, -4)\)
   (D) None
   (E) None
   (F) \(y = x - 4\)

Example 3: Find an equation of a rational function \( R(x) \) such that

(A) Vertical Asymptote: \( x = 3 \)

(B) Horizontal Asymptote: \( y = -1 \)

(C) \(x\)-intercept: 5

\[ R(x) = \frac{-(x - 5)}{(x - 3)} \]
EXERCISE 4-3

Consider the following function to find the following.

(A) Find the domain.
(B) Find hole if any
(C) Find intercepts
(D) What is the equation of the vertical asymptote(s) of this function?
(E) What is the equation of the horizontal asymptote(s) of this function?
(F) What is the equation of the oblique asymptote(s) of this function?

1 \( f(x) = \frac{x-2}{x^2-x-12} \) \hspace{1cm} 2 \( f(x) = \frac{x-2}{x^2+4x-12} \)

3 \( f(x) = \frac{x^2-x-6}{x^2-x-30} \) \hspace{1cm} 4 \( f(x) = \frac{x^2-x-6}{x^2+3x-18} \)
5 \quad f(x) = \frac{2x-3}{x+1} \\
6 \quad f(x) = \frac{x^2-x-6}{x-3} \\

7 \quad f(x) = \frac{x^2+x-12}{x+2} \\
8 \quad f(x) = \frac{x^2+2x-8}{x-3} \\

9 \quad f(x) = \frac{x-3}{x^2-4x+3}
Lecture 4-4 Inequalities

Interval Notation:

<table>
<thead>
<tr>
<th>NOTATION</th>
<th>INEQUALITY</th>
<th>GRAPH</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(a, b)$</td>
<td>$a &lt; x &lt; b$</td>
<td>$a\quad b$</td>
</tr>
<tr>
<td>$(a, b]$</td>
<td>$a &lt; x \leq b$</td>
<td>$a\quad b$</td>
</tr>
<tr>
<td>$[a, b)$</td>
<td>$a \leq x &lt; b$</td>
<td>$a\quad b$</td>
</tr>
<tr>
<td>$[a, b]$</td>
<td>$a \leq x \leq b$</td>
<td>$a\quad b$</td>
</tr>
<tr>
<td>$(a, \infty)$</td>
<td>$a &lt; x$</td>
<td>$a\quad b$</td>
</tr>
<tr>
<td>$[a, \infty)$</td>
<td>$a \leq x$</td>
<td>$a\quad b$</td>
</tr>
<tr>
<td>$(-\infty, a)$</td>
<td>$x &lt; a$</td>
<td>$a\quad b$</td>
</tr>
<tr>
<td>$(-\infty, a]$</td>
<td>$x \leq a$</td>
<td>$a\quad b$</td>
</tr>
</tbody>
</table>

Properties of Inequality

(A) Transitivity:
\[ a < b, \quad b < c \Rightarrow a < c \]

(B) Operations

<table>
<thead>
<tr>
<th></th>
<th>$a &lt; b$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Addition</td>
<td>$a + c &lt; b + c$</td>
</tr>
<tr>
<td>Subtraction</td>
<td>$a - c &lt; b - c$</td>
</tr>
</tbody>
</table>
| Multiplication | \[
\begin{cases}
ac < bc, & c > 0 \\
ac > bc, & c < 0
\end{cases}
\]
| Division | \[
\begin{cases}
\frac{a}{c} < \frac{b}{c}, & c > 0 \\
\frac{a}{c} > \frac{b}{c}, & c < 0
\end{cases}
\] |
Example 1: Solve the inequalities

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
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</thead>
<tbody>
<tr>
<td>(A)</td>
<td>$x^2 + x &lt; 6$</td>
<td>(B)</td>
</tr>
<tr>
<td>(A)</td>
<td>$(-3, 2)$</td>
<td>(B)</td>
</tr>
<tr>
<td>(C)</td>
<td>$[-4, 4]$</td>
<td>(D)</td>
</tr>
<tr>
<td>(E)</td>
<td>$(-\infty, -3) \cup (4, \infty)$</td>
<td></td>
</tr>
<tr>
<td>(F)</td>
<td>$\emptyset$</td>
<td></td>
</tr>
<tr>
<td>(G)</td>
<td>$(-\infty, \infty)$</td>
<td></td>
</tr>
<tr>
<td>(H)</td>
<td>$(2)$</td>
<td></td>
</tr>
<tr>
<td>(I)</td>
<td>$\left[ \frac{1}{2}, 2 \right]$</td>
<td></td>
</tr>
<tr>
<td>(J)</td>
<td>$(-\infty, 3) \cup (3, \infty)$</td>
<td></td>
</tr>
</tbody>
</table>

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<thead>
<tr>
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<tbody>
<tr>
<td>(C)</td>
<td>$16 - x^2 \leq 0$</td>
<td>(D)</td>
</tr>
</tbody>
</table>

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<tbody>
<tr>
<td>(E)</td>
<td>$x^2 - x \geq 12$</td>
<td>(F)</td>
</tr>
</tbody>
</table>

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</thead>
<tbody>
<tr>
<td>(G)</td>
<td>$x^2 - 6x + 13 &gt; 0$</td>
<td>(H)</td>
</tr>
</tbody>
</table>

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<thead>
<tr>
<th></th>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>(I)</td>
<td>$2x^2 - 5x + 2 \leq 0$</td>
<td>(J)</td>
</tr>
</tbody>
</table>
Example 2: Solve the inequalities

(A) \( x^3 \geq 4x \)
(B) \( x^3 < 9x \)

\[\text{(A)} \ [0, 2] \cup [4, \infty)\]
\[\text{(B)} \ (-\infty, -3) \cup (0, 3)\]

(C) \( x^3 - 2x^2 \geq 8x \)
(D) \( x^3 < x^2 + 12x \)

\[\text{(C)} \ [0, 2] \cup [4, \infty)\]
\[\text{(D)} \ (-\infty, -3) \cup (0, 4)\]

(E) \( x^3 > 8 \)
(F) \( (x - 2)^2(x + 1)(x + 3) \leq 0 \)

\[\text{(E)} \ (2, \infty)\]
\[\text{(F)} \ [-3, -1] \cup [2]\]
Example 3: Solve the inequalities

(A) \( \frac{x-4}{x+3} \leq 0 \)  
(B) \( \frac{x+4}{x+2} \geq 0 \)  
(C) \( \frac{x+3}{x-6} > 0 \)  
(D) \( \frac{x-5}{x+3} < 0 \)  
(E) \( \frac{x-2}{(x-3)(x+4)} \geq 0 \)  
(F) \( \frac{x-3}{x^2-3x-10} \leq 0 \)  
(G) \( \frac{x-3}{(x+2)^2(x-1)} \leq 0 \)  
(H) \( \frac{(x-2)^2}{(x+1)(x+3)} \geq 0 \)
EXERCISE 4-4

Solve the inequalities;

(A) \( x^2 - 3x \leq 18 \)  
(B) \( x^2 \geq 9 \)

(C) \( x^2 - 10x + 24 > 0 \)  
(D) \( 2x^2 - 9x + 4 \leq 0 \)

(E) \( x^3 - 16x \geq 0 \)  
(F) \( \frac{x-5}{x+3} \leq 0 \)

(G) \( \frac{x+7}{x-3} \geq 0 \)  
(H) \( \frac{x+5}{x^2-x-6} \leq 0 \)
PRACTICE PROBLEMS FOR UNIT 4

1. Form a polynomial of degree 6 with zeros: $-2$ of multiplicity 3, 1 of multiplicity 2, and 3 of multiplicity 1.

2. Find a polynomial function with real coefficients that Degree 4; zeros: $3, -1, 3 - 2i$

3. Find quotient and remainder
   (A) Divide $3x^3 - 17x^2 + 15x - 25$ by $x - 5$
   (B) Divide $3x^4 - 7x^2 - 11x + 5$ by $x - 1$.

4. Find all real/complex zeros of the polynomial
   (A) $f(x) = x^4 + x^3 - 14x^2 - 2x + 24$
   (B) $f(x) = x^4 + 6x^2 + 6x^2 - 24x - 40$.

5. Find all zeros of the polynomial and write the real zeros to factor completely
   (A) $f(x) = 3x^4 + 5x^3 - 17x^2 - 13x + 6$
   (B) $f(x) = 2x^4 + 7x^3 + x^2 - 7x - 3$.

6. Which of following is a factor of a polynomial $2x^4 + 7x^3 - 26x^2 - 49x + 30 = 0$?
   (A) $x - 5$         (B) $2x + 1$         (C) $x + 2$         (D) $x + 3$

7. Using the following rational function, answer the question.
   a) Find the domain of $g(x)$.
   b) Find the x-intercept(s) of $g(x)$ if any
   c) Find the y-intercept(s) of $g(x)$ if any
   d) Find the hole if any
   e) Find the vertical asymptote of $g(x)$ if any.
   f) Find the horizontal asymptote of $g(x)$ if any.
   g) Find the oblique asymptote of $g(x)$ if any.

   (A) $f(x) = \frac{x^2 - 6x - 27}{x + 3}$  (B) $f(x) = \frac{x^2 - x - 12}{x^2 + x - 6}$
   (C) $f(x) = \frac{x + 2}{x^2 + 2x - 8}$  (D) $f(x) = \frac{2x^2 - 7x + 3}{x - 2}$
   (E) $f(x) = \frac{x^2 + 2x - 15}{x^2 - x - 6}$  (F) $f(x) = \frac{x - 4}{x^2 - 4}$

1. $f(x) = (x + 2)^2(x - 1)^3(x - 3)$
2. $f(x) = (x - 3)(x + 1)(x^2 = 6x + 13)$
3. (A) $Q: 3x^2 - 2x + 5, R: 0$
   (B) $Q: 3x^4 + 3x^2 - 4x - 15, R: -10$

4. (A) $\pm \sqrt{2}, -4, 3$
   (B) $-2.2, -3 \pm i$

5. (A) $-3, -1, \frac{1}{2}, 2; f(x) =$
   $(x + 3)(x + 1)(3x - 1)(x - 2)$
   (B) $-3, -1, \frac{1}{2}, 1; f(x) =$
   $(x + 3)(x + 1)(2x + 1)(x - 1)$

6. C
7. (A) $(a)(-\infty, -3) \cup (-3, \infty)$
   (b) $g
   (c) - 9$
   (d) $-3$
   (e) None
   (f) None
   (g) $y = 3$
   (h) None
   (C) $(a)(-\infty, -4) \cup (-4, 2) \cup (2, \infty)$
   (b) $-2 (c) - \frac{1}{4}$
   (d) None
   (e) None
   (f) $y = 0 (g) None$
   (D) $(a)(-\infty, 2) \cup (2, \infty)$
   (b) $\frac{1}{2} (c) - \frac{3}{2}$
   (d) None
   (e) $y = 2$ (f) None
   (g) $y = 3$
   (E) $(a)(-\infty, -2) \cup (-2.3) \cup (3, \infty)$
   (b) $-5 (c) \frac{5}{2}$
   (d) $3 (e) x = -2$
   (f) $y = 1 (g) None$
   (F) $(a)(-\infty, -2) \cup (-2.2) \cup (2, \infty)$
   (b) $4 (c) 1$
   (d) None
   (e) $x = 2, x = -2$
   (f) $y = 0 (g) None$
DEFINITION: The composition of the function \( g \) with \( f \) is denoted by \( g \circ f \) and is defined by the equation

\[
(g \circ f)(x) = g(f(x))
\]

The domain of the composite function of \( g \circ f \) is the set of all \( x \) such that \( x \) is in the domain of \( f \) and \( f(x) \) is in the domain of \( g \).

Example 1: Let \( f(x) = 3x^2 - 5x + 8 \), \( g(x) = 2x \), and \( h(x) = x - 2 \). Find the following

<table>
<thead>
<tr>
<th></th>
<th>( (f \circ g)(3) )</th>
<th>( (g \circ f)(1) )</th>
<th>( (f \circ g)(-1) )</th>
<th>( (f \circ g)(x) )</th>
<th>( (g \circ f)(x) )</th>
<th>( (f \circ h)(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(A)</td>
<td>86</td>
<td>0</td>
<td>30</td>
<td>12 ( x^2 - 10x + 8 )</td>
<td>6( x^2 - 10x + 16 )</td>
<td>3( x^2 - 17x + 30 )</td>
</tr>
<tr>
<td>(B)</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(C)</td>
<td>30</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(D)</td>
<td>12 ( x^2 - 10x + 8 )</td>
<td></td>
<td></td>
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<td></td>
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</tr>
<tr>
<td>(E)</td>
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<td>(F)</td>
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</tbody>
</table>
Example 2: Let \( f(x) = 2x^2 - 7x + 3 \), \( g(x) = 3x \), and \( h(x) = x - 3 \). Find the following

<table>
<thead>
<tr>
<th></th>
<th>(A) ((f \circ g)(1))</th>
<th>(B) ((g \circ f)(1))</th>
<th>(C) ((f \circ g)(-1))</th>
<th>(D) ((f \circ g)(x))</th>
<th>(E) ((g \circ f)(x))</th>
<th>(F) ((f \circ h)(x))</th>
</tr>
</thead>
<tbody>
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</tr>
<tr>
<td></td>
<td>(A) 0</td>
<td>(B) -6</td>
<td>(C) 42</td>
<td>(D) (12x^2 - 21x + 3)</td>
<td>(E) (6x^2 - 21x + 9)</td>
<td>(F) (2x^2 - 19x + 42)</td>
</tr>
</tbody>
</table>

Example 3: Let \( f(x) = \frac{1}{x-1} \), and \( g(x) = \frac{4}{x} \). Find the domain of the following function

<table>
<thead>
<tr>
<th></th>
<th>(A) ((g \circ f)(x))</th>
<th>(B) ((f \circ g)(x))</th>
</tr>
</thead>
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<tr>
<td></td>
<td>(A) ((-\infty, 1) \cup (1, \infty))</td>
<td>(A) ((-\infty, 0) \cup (0, 4) \cup (4, \infty))</td>
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<td>(B) ((-\infty, 0) \cup (0, 4) \cup (4, \infty))</td>
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ONE TO ONE FUNCTIONS

One to one function (Invertible function): A function for which every element of the range of the function corresponds to exactly one element of the domain.

HORIZONTAL LINE TEST (One to one function or not):
A test uses to determine if a function is one-to-one. If a horizontal line intersects a function's graph more than once, then the function is not one-to-one.

Example 4: Decide whether the following is a one to one function or not.

(A) \( f(x) = |x + 5| - 2 \)  
(B) \( y = x^3 - 2 \)  
(C) \( y = x^2 + 2x - 3 \)  
(D) \( y = \frac{x-1}{x+2} \)  
(E) \( x^2 + y^2 = 9 \)

Example 5: Find the inverse of the following function

(A) \( f(x) = \sqrt[3]{x + 2} \)  
(B) \( f(x) = \sqrt[3]{x - 3} \)  
(C) \( f(x) = x^3 + 2 \)  
(D) \( f(x) = x^5 - 5 \)
Example 6: Find the inverse of the following function

(A) \( f(x) = (x - 5)^3 + 8 \)  
(B) \( f(x) = \sqrt[5]{x} + 6 \)  

\[
\begin{align*}
(A) \quad f^{-1}(x) &= \sqrt[5]{x} - 8 + 5 \\
(B) \quad f^{-1}(x) &= (x - 6)^5 \\
(C) \quad f^{-1}(x) &= \frac{5}{x} \\
(D) \quad f^{-1}(x) &= \frac{3x - 2}{3x + 1}
\end{align*}
\]

(C) \( f(x) = \frac{5}{x} - 3 \)  
(D) \( f(x) = \frac{x - 2}{3x + 4} \)

Example 5: Given the graph of \( y = f(x) \), draw the graph of \( y = f^{-1}(x) \).
EXERCISE 5-1.

1. Let \( f(x) = 5x^2 - 3x + 9 \) and \( g(x) = 2x \)
   (A) \((f \circ g)(2)\)  (B) \((g \circ f)(-3)\)

   (C) \((f \circ g)(x)\)  (D) \((g \circ f)(x)\)

2. Decide whether the following is a one to one function or not.
   (A) \( y = x^2 - 4x + 5 \)  (B) \( y = x^3 - 2 \)

   (C) \( y = |x + 1| \)  (D) \( y = x^3 - 4x \)

3. Find the inverse function of \( f \).
   (A) \( f(x) = x^3 + 5 \)  (B) \( f(x) = \sqrt[3]{x} - 1 \)
Example 1: Write each equation in its equivalent exponent form

(A) \( 2 = \log_5(x) \)  
(B) \( 3 = \log_b(64) \)  
(C) \( \log_5(b) = a \)  
(D) \( \log_3(x - 3) = r \)

\( x = 5^2 \)  
\( 64 = b^3 \)  
\( b = 5^a \)  
\( x - 3 = 3^r \)

Example 2: Write each equation in its equivalent logarithmic form.

(A) \( 12^2 = x \)  
(B) \( e^{12} = h \)  
(C) \( a^5 = b \)  
(D) \( (5 - x)^4 = b \)

\( 12 = \log_{12}(x) \)  
\( 12 = \ln(h) \)  
\( 5 = \log_a(b) \)  
\( 4 = \log_{5-x}(b) \)

Example 3: Evaluate each of the following logarithms.

(A) \( \log_2(16) \)  
(B) \( \log_5(7) \)  
(C) \( \log_2(10) \)  
(D) \( \log_7(6) \)

\( 4 \)  
\( 1.2091 \)  
\( 3.3219 \)  
\( 0.9208 \)

Example 4: Find the exact solution of the following equation.

(A) \( e^x = 5 \)  
(B) \( 2^{x-3} = 5 \)  
(C) \( 300e^{0.5x} = 900 \)  
(D) \( 10^{x+3} = 7 \)

\( \ln 5 \)  
\( 3 + \log_2(5) \)  
\( 2 \ln(3) \)  
\( -3 + \log(7) \)
Example 5: Solve the equation

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<tbody>
<tr>
<td>(A)</td>
<td>3^{x+1} = 27</td>
<td>(B)</td>
<td>2^{x-3} = 16</td>
<td>(C)</td>
<td>4^{5x-7} = 8</td>
<td>(D)</td>
</tr>
<tr>
<td>(E)</td>
<td>9^{5x-2} = 27^{x+1}</td>
<td>(F)</td>
<td>4^{3x-2} = 8^{3-x}</td>
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<td></td>
<td></td>
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<tr>
<td>(G)</td>
<td>5^{x-2} = \frac{1}{125}</td>
<td>(H)</td>
<td>\left(\frac{1}{2}\right)^x = 32</td>
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<td></td>
<td></td>
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<tr>
<td>(I)</td>
<td>4^x = 2^{x+2}</td>
<td>(J)</td>
<td>e^{x^2+6} = e^{5x}</td>
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<tr>
<td>(K)</td>
<td>\left(\frac{3}{2}\right)^{x+2} = \left(\frac{1}{5}\right)^3</td>
<td>(L)</td>
<td>e^{x^2-3} = e^{2x}</td>
<td></td>
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</table>
EXERCISE 5-2

1. Write each equation in its equivalent exponent form
   (A) \( x = \log_a(b) \)  
   (B) \( y = \log_b(x + 5) \)

2. Write each equation in its equivalent logarithmic form.
   (A) \((x + 2)^2 = y\)  
   (B) \(a^{x+3} = y\)

3. Solve the equation
   (A) \(3^{5x-4} = 81\)  
   (B) \(4^{3x-7} = 8^{x-6}\)

   (C) \(2^{x+4} = \frac{1}{16}\)  
   (D) \(\left(\frac{3}{2}\right)^{2x-3} = \frac{4}{9}\)
Example 1: Use logarithmic properties to expand each expression as much as possible. Assume that all variables are positive.

(A) \( \log_a(u^6v^5) \)  
(B) \( \log_b(x^2\sqrt{y}) \)  
(C) \( \ln\left(\frac{x}{wyz}\right) \)  
(D) \( \log\left(\frac{x^3y}{w^2}\right) \)

Example 2: Write each expression as a single logarithm.

(A) \( \log_{10} x + \log_{10} y \)  
(B) \( 3 \ln(x) - 2 \ln(y) \)  
(C) \( 3 \log(x) - 4 \log(t) \)  
(D) \( 4 \log(y) + \log(x^4y^2) \)  
(E) \( 2 \ln x - 3 \ln y + 4 \ln z \)  
(F) \( 5 \log(x) - 3 \log(y) - 2 \log(z) \)
Example 3: Solve the equation:

(A) \( \log_4(x - 3) = \log_4(4) \)  
(B) \( \log_3(x + 1) = \log_3(3x + 7) \)

(C) \( \ln(x + 3) = \ln(2x) \)  
(D) \( \log(5x - 4) = \log(3x + 7) \)

(E) \( \log_2(x + 5) = 3 \)  
(F) \( \log_3(2x - 5) = 2 \)

(G) \( \log_2(6x + 2) = 4 \)  
(H) \( \log_6(4x - 2) = 1 \)

(I) \( \log(x + 3) = 2 \)  
(J) \( \log_5(3x - 2) = 2 \)
Example 4: Solve the equation:

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<tbody>
<tr>
<td>(A)</td>
<td>(\log_2(x) + \log_2(x + 5) = \log_2(14))</td>
<td>(B)</td>
</tr>
<tr>
<td>(A)</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>(B)</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>(C)</td>
<td>No solution</td>
<td></td>
</tr>
<tr>
<td>(D)</td>
<td>11</td>
<td></td>
</tr>
<tr>
<td>(E)</td>
<td>No solution</td>
<td></td>
</tr>
<tr>
<td>(F)</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>(G)</td>
<td>5</td>
<td></td>
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<tr>
<td>(H)</td>
<td>(-5.5)</td>
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\(\log_2(x) - \log_2(4) = \log_2(x + 2)\) \hspace{1cm} \(\log(x - 2) + \log(3) = \log(2x + 5)\)

\(\ln(2) + \ln(x - 1) = \ln(3x)\) \hspace{1cm} \(\log_3(x) + \log_3(x + 6) = 3\)

\(\log(x) + \log(x - 3) = 1\) \hspace{1cm} \(\log_{12}(13 - x) + \log_{12}(13 + x) = 1\)
EXERCISE 5-3

1. Use logarithmic properties to expand each expression as much as possible. Assume that all variables are positive.
   (A) \( \log_a(x^3y^4) \)  
   (B) \( \log_b \left( \frac{x^5}{y^7} \right) \)

2. Write each expression as a single logarithm.
   (A) \( 5 \log_{10} x + 9 \log_{10} y \)  
   (B) \( 4 \ln(x) - 5 \ln(y) \)

3. Solve the equation.
   (A) \( \log_2(4x - 5) = \log_2(2x + 4) \)  
   (B) \( \ln(x^2) = \ln(2x + 3) \)
   (C) \( \log_2(3x - 2) = 4 \)  
   (D) \( \log(2x - 1) = 2 \)
LECTURE 5.4 COMPOUNDED INTEREST AND APPLICATION

**COMPOUND INTEREST:** After \( t \) years, the balance \( A \) in an account with principal \( P \) and annual interest rate \( r \) (in decimal form) is given by the following formulas:

- If the interest is compounded annually,

<table>
<thead>
<tr>
<th>Time</th>
<th>Formula</th>
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<tbody>
<tr>
<td>After 1 year</td>
<td>( A = P + Pr = P(1 + r) )</td>
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<tr>
<td>After 2 years</td>
<td>( A = P(1 + r) + [P(1 + r)]r = P(1 + r)(1 + r) = P(1 + r)^2 )</td>
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<tr>
<td>After 3 years</td>
<td>( A = P(1 + r)^2 + [P(1 + r)^2]r = P(1 + r)^2(1 + r) = P(1 + r)^3 )</td>
</tr>
<tr>
<td>( \vdots )</td>
<td>( \vdots )</td>
</tr>
<tr>
<td>After ( t ) years</td>
<td>( A = P(1 + r)^t )</td>
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</table>

- \( n \) compounding per year case: it is computed \( n \) times per a year. It means that each period interest is \( \frac{r}{n} \% \) and it is computed \( n \times t \) times for \( t \) years.

\[
A = P \left(1 + \frac{r}{n}\right)^{n \times t}
\]

- Continuously compounded case: it is the approaching balance as \( n \) approached \( \infty \).

\[
A = Pe^{rt}
\]

Example 1: A total of $1,000 is invested at an annual interest rate of 2.5%, compounded monthly. Find the balance in the account after 5 years. $1133.00

Example 2: A total of $7,000 is invested at an annual interest rate of 4.5%, compounded monthly. Find the balance in the account after 8 years. $10,026.55
Example 3: A total of $5,000 is invested at an annual interest rate of 5%, compounded quarterly. Find the balance in the account after 10 years.

$8218.10

Example 4: A total of $5,000 is invested at an annual interest rate of 5%, compounded continuously. Find the balance in the account after 10 years.

$8243.61

Example 5: The number, $A$, of bacteria found in a culture is a function of time, $t$, in minutes and is given by the formula $A = 1500e^{0.5t}$ with the initial number of bacteria $A_0 = 1500$. After how many minutes will there be three times the initial number of bacteria? Round to 4 decimal places.

2.1972

Example 6: If $3600 is invested at 10% compounded quarterly how many is in the account after 20 years? Round your answer to the nearest dollar.

$25,954

Example 7: How long does it take money to triple if it is invested at 15% compounded continuous?

7.32 years
1. Rewrite the following in logarithmic form
   (A) $2^4 = 16$  
   (B) $3^2 = 9$  
   (C) $x^a = b$  
   (D) $(x - 2)^a = y$

2. Rewrite the following in exponential form
   (A) $3 = \log_2(8)$  
   (B) $2 = \log_5(25)$  
   (C) $a = \log_z(x)$  
   (D) $x = \log(y)$

3. Evaluate the following. (Give an approximation to four decimal places.)
   (A) $\log_2(5)$  
   (B) $\log_6(11)$

4. Find $f^{-1}(x)$ for the one-to-one function $f(x)$.
   (A) $f(x) = x^3 - 8$  
   (B) $f(x) = x^3 + 2$  
   (C) $f(x) = \sqrt[3]{x} + 4$  
   (D) $f(x) = \frac{3}{\sqrt{x} - 5}$

5. Decide whether the following is a one to one function or not.
   (A) $y = x^3 + 2$  
   (B) $y = x^2 - 4$  
   (C) $y = \frac{x - 1}{x + 2}$  
   (D) $y = x^3 - 9x$  
   (E) $y = \sqrt{x} + 2$  
   (F) $x^2 + y^2 = 9$

6. Solve the following equations.
   (A) $3^{2x-1} = 81$  
   (B) $9^{2x-8} = 27^{x-4}$  
   (C) $25^{2x-6} = \left(\frac{1}{125}\right)^x$  
   (D) $\left(\frac{5}{7}\right)^x = \left(\frac{2}{3}\right)^{2x-24}$  
   (E) $\ln(2 + \ln(3x - 5)) = \ln(32)$  
   (F) $\log(6x - 2) = 2$  
   (G) $\log(x) + \log(x - 21) = 2$  
   (H) $\log_3(x) + \log_3(x - 6) = 3$

7. If the values of $\log_a(2) = 0.2038$ and $\log_a(3) = 0.3230$, find the following
   (A) $\log_a(24)$  
   (B) $\log_a(\sqrt{3})$  
   (C) $\log_a\left(\frac{3}{2}\right)$  
   (D) $\log_a\left(\frac{8}{9}\right)$

8. A total of $10,000$ is invested at an annual interest rate of $2.5\%$, compounded monthly. Find the balance in the account after 10 years.

9. The number, $A$, of bacteria found in a culture is a function of time, $t$, in minutes and is given by the formula $A = 2000e^{0.5t}$ with the initial number of bacteria $A_0 = 2000$. After how many minutes will there be double the initial number of bacteria? Round to 4 decimal places.

10. The exponential growth model $A = 30e^{0.15t}$ describes the population of a city in the United States, in thousands, $t$ years after 1994. Use this model to solve the following:
    (A) What was the population of the city in 2000?
    (B) What will the population of the city be in 2005?
PRACTICE PROBLEMS FOR FINAL

(1) Function or not (vertical line test or \( y = x \) expression)

1. Find all relations which are functions.
   (A) \( x^2 + y^2 = 4 \)  
   (B) \( y = \frac{3x-1}{x+2} \)  
   (C) \( |y| = x - 3 \)  
   (D) None of above

(2) One to one function (Horizontal line test)

2. Find all one to one functions.
   (A) \( f(x) = x^2 - 4x \)  
   (B) \( f(x) = x^3 - 9x \)  
   (C) \( f(x) = |x + 5| \)  
   (D) \( f(x) = x^3 - 4 \)

(3) Three symmetry
   y-axis: even  
   Origin : odd

3. Which of the following function is symmetric with respect to the origin?
   (A) \( f(x) = x^3 + 4 \)  
   (B) \( f(x) = x^4 - 1 \)  
   (C) \( f(x) = x^3 - 4x \)  
   (D) \( f(x) = x^2 + 2x \)

4. Which of the following function is symmetric with respect to the y-axis?
   (A) \( f(x) = x^3 - 9x \)  
   (B) \( f(x) = x^4 - 1 \)  
   (C) \( f(x) = \frac{3x}{x^2+9} \)  
   (D) \( f(x) = x^2 + 2x \)

(4) Find the value of a function.

5. Evaluate \( f(-3) \) if \( f(x) = 3x^2 - 2x + 1 \)
   (A) 22  
   (B) 34  
   (C) -20  
   (D) -34

6. Evaluate \( f(-2) \) if \( f(x) = \frac{x-3}{x^2+1} \)
   (A) -1  
   (B) \( -\frac{5}{6} \)  
   (C) \( \frac{5}{3} \)  
   (D) 1

7. Evaluate \( f(2) \) if \( f(x) = \begin{cases} 
  x^2, & x < 0 \\
  1, & x = 0 \\
  2x + 1, & x > 0 
\end{cases} \)
   (A) 4  
   (B) 10  
   (C) 1  
   (D) 5

(5) Domain of a function.

8. Find the domain of the following function \( f(x) = \log(x + 2) \)
   (A) \( (-2, \infty) \)  
   (B) \( [-2, \infty) \)  
   (C) \( (-\infty, -2) \cup (-2, \infty) \)  
   (D) \( (-\infty, \infty) \)

9. Find the domain of the following function \( f(x) = \sqrt{3x - 12} \)
   (A) \( (-\infty, 4] \)  
   (B) \( [4, \infty) \)  
   (C) \( (-\infty, -4] \cup [4, \infty) \)  
   (D) \( (-\infty, \infty) \)

10. Find the domain of \((f + g)(x)\) where \( f(x) = 1 + \frac{1}{x} \) and \( g(x) = \frac{1}{x} \)
    (A) \( (-\infty, 0) \)  
    (B) \( (0, \infty) \)  
    (C) \( (-\infty, 0) \cup (0, \infty) \)  
    (D) \( (-\infty, \infty) \)
(6) Find the intercepts
11. Identify the intercepts of the following function: \( f(x) = \frac{2x+6}{x+2} \)
   (A) \( x\)-int: \((-3,0)\); \( y\)-int: \(0,-2)\)  
   (B) \( x\)-int: \((-3,0)\); \( y\)-int: \(0,3)\)  
   (C) \( x\)-int: \((0,-3)\); \( y\)-int: \((-2,0)\)  
   (D) \( x\)-int: \((0,-3)\); \( y\)-int: \((3,0)\)
12. Identify the intercepts of the following function: \( f(x) = x^2 + x - 12 \)
   (A) \( x\)-int: \((3,0),(-4,0)\); \( y\)-int: \(0,-12)\)  
   (B) \( x\)-int: \((-12,0)\); \( y\)-int: \((0,3)\)  
   (C) \( x\)-int: \((-3,0),(4,0)\); \( y\)-int: \(0,-12)\)  
   (D) \( x\)-int: \((3,0),(-4,0)\); \( y\)-int: \(0,12)\)

(7) Factor and x-intercept
13. Which of the following is a factor of a polynomial \( f(x) = 6x^3 - 35x^2 + 19x + 30? \)
   (A) \( x + 5 \)  
   (B) \( 2x + 3 \)  
   (C) \( 3x + 2 \)  
   (D) \( x + 6 \)
14. Which of the following is a factor of \( f(x) = 2x^4 - 7x^3 - 2x^2 + 13x + 6? \)
   (A) \( x - 5 \)  
   (B) \( x + 2 \)  
   (C) \( 2x + 1 \)  
   (D) \( x + 3 \)

(8) Operations of functions
15. Let \( f(x) = x^2 - 6x \) and \( g(x) = \sqrt{x+9} \). Which of the following is true?
   (A) \( (f + g)(-1) = 5 \)  
   (B) \( (f \cdot g)(0) = -3 \)  
   (C) \( (f - g)(-5) = 53 \)  
   (D) \( \left(\frac{f}{g}\right)(7) = 1 \)

(9) Composition
16. Find the composite function \( (f \circ g)(x) \) if \( f(x) = 2x^2 - 4x - 1 \) and \( g(x) = 3x \)
   (A) \( 6x^3 - 12x^2 - 3x \)  
   (B) \( 36x^2 - 12x - 1 \)  
   (C) \( 6x^2 - 12x - 1 \)  
   (D) \( 18x^2 - 12x - 1 \)
17. Find \( (f \circ g)(-1) \) if \( f(x) = 2x^2 - 3x + 7 \) and \( g(x) = 2x - 1 \)
   (A) \(-2 \)  
   (B) \(34 \)  
   (C) \(-4x^3 + 8x^2 - 17x + 7 \)  
   (D) \(4x^2 - 2x + 5 \)

(10) Inverse functions
18. Find the inverse of the function \( f(x) = \frac{3}{\sqrt{x} + 8} \)
   (A) \( f^{-1}(x) = x^3 + 8 \)  
   (B) \( f^{-1}(x) = \frac{1}{\sqrt{x} + 8} \)  
   (C) \( f^{-1}(x) = x^3 - 8 \)  
   (D) \( f^{-1}(x) = x \)  
19. Find the inverse of the function \( f(x) = \frac{2x-3}{4x+1} \)
   (A) \( f^{-1}(x) = \frac{4x+1}{2x-3} \)  
   (B) \( f^{-1}(x) = \frac{2x-3}{4x+1} \)  
   (C) \( f^{-1}(x) = \frac{4x-1}{2x+3} \)  
   (D) \( f^{-1}(x) = \frac{4x+1}{2x-3} \)
20. Describe in a sentence when \( y = x^3 - 3 \) is the inverse function of \( y = \frac{3}{\sqrt{x} + 3} \)
   (A) Mirror image about x-axis  
   (B) Mirror image about y-axis  
   (C) Mirror image about the origin  
   (D) Mirror image about y = x
(11) Solve a polynomial

21. Solve the equation $x(x + 6) = 3$
   - (A) $-6 \pm \sqrt{3}$
   - (B) $-6 \pm 2\sqrt{3}$
   - (C) $-3 \pm 2\sqrt{3}$
   - (D) $3 \pm 2\sqrt{3}$

22. Solve the equation $2x^2 - 4x + 1 = 0$
   - (A) $\frac{-2\pm\sqrt{2}}{2}$
   - (B) $\frac{2\pm\sqrt{2}}{2}$
   - (C) $\frac{2\pm i\sqrt{2}}{2}$
   - (D) $1\pm\sqrt{2}$

23. Solve the equation: $(x - 2)^2 = 18$
   - (A) $-2 \pm 2\sqrt{3}$
   - (B) $2 \pm 2\sqrt{3}$
   - (C) $2 \pm 3\sqrt{2}$
   - (D) $3 \pm 2i\sqrt{3}$

24. Solve the equation: $4x^2 - 2x + 3 = 0$
   - (A) $\frac{1 \pm i\sqrt{11}}{2}$
   - (B) $\frac{1 \pm i\sqrt{11}}{4}$
   - (C) $\frac{-1 \pm i\sqrt{11}}{4}$
   - (D) $\frac{-1 \pm i\sqrt{11}}{2}$

25. Solve the equation: $3x^3 - 5x^2 - 2x = 0$
   - (A) $-\frac{1}{3}, 0, 2$
   - (B) $-\frac{1}{3}, 2$
   - (C) $\frac{1}{3}, 0, -2$
   - (D) $\frac{1}{3}, -2$

26. Solve the equation: $x^3 - 2x^2 - 9x + 18 = 0$
   - (A) $-2, 2, 9$
   - (B) $-3, -2, 3$
   - (C) $-3, 2, 3$
   - (D) $-9, -2, 2$

(12) Final all real/complex zeros

27. Find the real/complex zeros of the function.
   - $f(x) = x^4 + 2x^3 + 22x^2 + 50x - 75$
   - (A) $-3, 1, -5, 5$
   - (B) $-3, 1, -5i, 5i$
   - (C) $3, -1, -5, 5$
   - (D) $3, -1, -5i, 5i$

28. Find the real/complex zeros of the function and write it as factor form.
   - $f(x) = x^4 - 5x^3 + 4x^2 + 22x - 20$
   - (A) $-2, 1, 3 \pm i$
   - (B) $2, -1, 3 \pm i$
   - (C) $-2, 1, -3 \pm i$
   - (D) $2, -1, -3 \pm i$

29. Find the real/complex zeros of the function and write it as factor form.
   - $f(x) = 2x^4 + 7x^3 + x^2 - 7x - 3$
   - (A) Zeros: $-\frac{1}{2}, -3, -1, 1$; $f(x) = (x + \frac{1}{2})(x + 3)(x + 1)(x - 1)$
   - (B) Zeros: $-\frac{1}{2}, -3, -1, 1$; $f(x) = (2x + 1)(x + 3)(x + 1)(x - 1)$
   - (C) Zeros: $-\frac{1}{2}, 3, -1, 1$; $f(x) = (2x - 1)(x - 3)(x + 1)(x + 1)$
   - (D) Zeros: $\frac{1}{2}, 3, -1, 1$; $f(x) = (2x + 1)(x + 3)(x - 1)(x + 1)$

30. Form a polynomial of degree 3 with zeros: $-2$ with multiplicity 2 and 4 with multiplicity 1.
   - (A) $(x - 2)^2(x + 4)$
   - (B) $(x - 2)^2(x + 1)^4$
   - (C) $(x + 2)^2(x - 4)$
   - (D) $(x + 2)^2(x + 1)^4$

31. Form a polynomial of degree 3 with zeros: $2, 4 \pm i$
   - (A) $x^3 - 10x^2 + 33x - 34$
   - (B) $x^3 - x^2 + 32x - 34$
   - (C) $x^3 - 10x^2 + 31x - 30$
   - (D) $x^3 + 10x^2 + 33x + 34$
32. Solve the equation: $\sqrt{10x - 1} = x + 2$
   (A) Only 1  (B) Only 5  (C) 1, 5  (D) $-1, -5$

33. Solve the equation: $\sqrt{x + 12} = x$
   (A) Only 4  (B) Only $-3$  (C) $4, -3$  (D) $3, -4$

34. Solve the equation: $\frac{5}{2x-3} = \frac{3}{x+5}$
   (A) 16  (B) $\frac{23}{2}$  (C) $\frac{61}{3}$  (D) 34

35. Solve the equation: $\frac{3}{x-3} + \frac{5}{x-4} = \frac{x^2-20}{x^2-7x+12}$
   (A) Only 1  (B) Only 7  (C) 1, 7  (D) Only $-1$

36. Find the equation of a line which contains two points $(-3, 5)$ and $(0, 1)$
   (A) $y = -\frac{4}{3}x + 1$  (B) $y = \frac{2}{3}x + 1$
   (C) $y = -\frac{3}{4}x + 1$  (D) $y = -\frac{3}{4}x + \frac{11}{4}$

37. Find the equation of a line which contains a point $(2, 4)$ and is parallel to $5x - y = 20$.
   (A) $y = 5x - 14$  (B) $y = 5x - 6$
   (C) $y = -5x + 14$  (D) $y = \frac{1}{5}x + 4$

38. Find the equation of a line which contains a point $(3, 4)$ and is perpendicular to $3x - 8y = -23$.
   (A) $y = -\frac{8}{3}x + 12$  (B) $y = -\frac{8}{3}x + 36$
   (C) $y = \frac{8}{3}x - 12$  (D) $y = \frac{8}{3}x - 4$

39. A truck rental company rents its trucks at a daily base price of $29 plus 39 cents for each mile the truck is driven. Mr. Park rented a truck for one day and was charged $165.5 when he returned the truck. How many miles he had driven the rental truck?
   (A) 200 miles  (B) 250 miles  (C) 350 miles  (D) 450 miles

40. Find the value of $y$ which satisfies the linear system of equations: $5x - y = 13$, $2x + 3y = 12$
   (A) $-1$  (B) 0  (C) 2  (D) 3

41. A restaurant manager wants to purchase 200 sets of dishes. One design costs $25 per set, while another costs $45 per set. If she only has $7400 to spend, how many of the $25 design should she buy?
   (A) 75  (B) 80  (C) 112  (D) 120

42. A bank loaned out $12,000, part of it at the rate of 8% per year and the rest at the rate of 18% per year. If the interest received totaled $1000, how much was loaned at 8%?
   (A) $400  (B) $5,500  (C) $6,500  (D) $11,600
43. There were 436 people at a civic club fundraiser. Members paid $4.5 per ticket and nonmembers paid $8.25 per ticket. If total receipts amounted to $2562, how many members attended the fundraiser?
(A) 264  (B) 276  (C) 160  (D) 172

(16) Quadratic functions
44. Find the vertex of \( y = 3x^2 - 12x + 3 \)
(A) \((2, -8)\)  (B) \((2, -9)\)  (C) \((-2, -9)\)  (D) \((2, -5)\)

45. Find the maximum/minimum of the quadratic function \( y = -2x^2 + 8x - 5 \)
(A) Maximum: \((2, -13)\)  (B) Maximum: \((2, 3)\)  (C) Minimum: \((-2, 3)\)  (D) Minimum: \((-2, -13)\)

46. Find the quadratic function such that its vertex is \((2, -1)\) and its \(y\)-intercept is \((0, 7)\).
(A) \(y = (x - 2)^2 - 1\)  (B) \(y = \frac{3}{2}(x - 2)^2 - 1\)  (C) \(y = 2(x - 2)^2 - 1\)  (D) \(y = \frac{3}{2}(x + 2)^2 + 1\)

47. The height, in feet, from the ground of a ball dropped from a 176-foot building \(t\) seconds after it is dropped is given by the formula \(H(t) = -16t^2 + 176\). How long does it take the ball to hit the ground?
(A) 16 seconds  (B) 5.5 seconds  (C) 3.3 seconds  (D) 11 seconds

48. The area of the opening of a rectangular window is 143 cm\(^2\). If the length is 2 cm more than the width, what is the width of the window?
(A) 9 cm  (B) 11 cm  (C) 12 cm  (D) 13 cm

49. The profit that the vendor makes per day by selling \(x\) pretzels is given by the function \(P(x) = -0.004x^2 + 2.8x - 200\). Find the number of pretzels that must be sold to maximize profit.
(A) 500  (B) 450  (C) 350  (D) 300

50. Beth has 3000 feet of fencing available to enclose a rectangular field. One side of the field lies along a river, so only three sides require fencing. Express the area \(A\) of the rectangle as a function of \(x\), where \(x\) is the length of the side parallel to the river. For what value of \(x\) is the area largest?
(A) 750 ft  (B) 1000 ft  (C) 1500 ft  (D) 2000 ft

(17) Inequality
51. Solve the inequality: \(x^2 \geq x + 6\)
(A) \((-\infty, -2] \cup [3, \infty)\)  (B) \((-\infty, -3] \cup [2, \infty)\)  (C) \([-2, 3]\)  (D) \((-\infty, \infty)\)

52. Solve the inequality: \(x^2 - x - 12 < 0\)
(A) \((-4, 3)\)  (B) \((-3, 4)\)  (C) \((-\infty, -3) \cup (4, \infty)\)  (D) \((-\infty, -4) \cup (3, \infty)\)

53. Solve the rational inequality: \(\frac{x-3}{x+2} \leq 0\)
(A) \((-2, 3]\)  (B) \((-3, 2]\)  (C) \((-2, 3)\)  (D) \((-\infty, -2) \cup [3, \infty)\)
54. Solve the inequality: \(\frac{x+1}{x-1} \leq 2\)
   (A) \((-\infty, 1) \cup [3, \infty)\)  \quad (B) \((-\infty, 1) \cup [5, \infty)\)
   (C) (1,3]  \quad (D) (1,5]

55. Find all vertical asymptotes for the function \(f(x) = \frac{x^2-1}{x^2+4x+3}\)
   (A) \(x = -1, x = -3\)  \quad (B) \(y = -1, y = -3\)
   (C) \(x = 1\)  \quad (D) \(x = -3\)

56. Find the horizontal asymptote for the function \(f(x) = \frac{x-1}{x^2-5x+6}\)
   (A) \(y = 2\)  \quad (B) \(y = 1\)
   (C) \(y = 0\)  \quad (D) No horizontal asymptote

57. Find the oblique asymptote for the function \(f(x) = \frac{x^2+2x-3}{x+3}\)
   (A) \(y = x - 1\)  \quad (B) \(y = 1\)
   (C) \(y = 0\)  \quad (D) No oblique asymptote

58. Find the remainder when \(f(x) = x^3 + 2x - 1\) is divided by \(x - 1\)
   (A) 0  \quad (B) 1  \quad (C) 2  \quad (D) -4

59. Find the approximate of \(\log_2(3)\). Round your answer to two decimal digits.
   (A) 5.88  \quad (B) 1.58
   (C) 1.10  \quad (D) 0.63

60. Rewrite the expression as a single logarithm form: \(2 \ln(x) - \ln(x+2) + 3 \ln(z)\)
   (A) \(\ln\left(\frac{x^2(x+2)}{z^3}\right)\)  \quad (B) \(\ln\left(\frac{x^2}{(x+2)z^3}\right)\)
   (C) \(\ln\left(\frac{x^2z^3}{(x+2)}\right)\)  \quad (D) \(\ln(x-2+3z)\)

61. Write the exponential equation \(2^a = x\) in the logarithmic form.
   (A) \(a = \log_2(x)\)  \quad (B) \(2 = \log_x(a)\)
   (C) \(2 = \log_a(x)\)  \quad (D) \(a = \log_x(2)\)

62. Solve the equation: \(3^{x-3} = \left(\frac{1}{9}\right)^{-x}\)
   (A) 6  \quad (B) 1  \quad (C) -3  \quad (D) -1

63. Solve the equation: \(2^{2x+1} = 4^{-x+2}\)
   (A) -1  \quad (B) \(\frac{1}{4}\)  \quad (C) \(\frac{1}{2}\)  \quad (D) \(\frac{3}{4}\)

64. Solve the equation: \(\ln(x) + \ln(x-1) = \ln(12)\)
   (A) 4, -3  \quad (B) 3, -4  \quad (C) Only 3  \quad (D) Only 4

65. Solve the equation: \(\log_2(x) + \log_2(x+2) = 3\)
   (A) -4, 2  \quad (B) Only -4  \quad (C) Only 2  \quad (D) Only 4
66. The number, \( A \), of bacteria found in a culture is a function of time, \( t \), in minutes and is given by the formula \[ A = 2500e^{0.5t} \] with the initial value \( A_0 = 2500 \). After how many minutes will there be double the initial amount of bacteria? Round to 2 decimal places.

(A) 1.39  
(B) 1.54  
(C) 2.34  
(D) 2.77

67. Find the distance between points \((-1,0)\) and \((2,4)\)

(A) 4  
(B) \( \sqrt{17} \)  
(C) 5  
(D) \( \sqrt{27} \)

68. Find the center and radius of the circle: \( x^2 + y^2 - 4x + 6y + 9 = 0 \)

(A) Center \((-2,3)\); radius 4  
(B) Center \((2,-3)\); radius 4  
(C) Center \((-2,3)\); radius 2  
(D) Center \((2,-3)\); radius 2

69. Find the equation of a circle whose two diameter end points are \((1,-5)\) and \((3,7)\).

(A) \((x - 2)^2 + (y - 1)^2 = 35\)  
(B) \((x - 2)^2 + (y - 1)^2 = 37\)  
(C) \((x + 2)^2 + (y - 1)^2 = 45\)  
(D) \((x + 2)^2 + (y - 1)^2 = 61\)

70. Find the accumulated value of an investment of $680 at 4% compounded monthly for 17 years. Round your answer to the nearest cent.

(A) $1142.40  
(B) $1340.72  
(C) $1273.63  
(D) $1324.57

71. If $3600 is invested at 10% compounded quarterly how many is in the account after 20 years? Round your answer to the nearest dollar.

(A) $27,345  
(B) $23,596  
(C) $25,954  
(D) $10,804,500

72. How long does it take money to triple if it is invested at 15% compounded continuous?

(A) 9.56 years  
(B) 9.86 years  
(C) 7.46 years  
(D) 7.32 years

73. The graph of \( y = x^3 \) is shifted right by 5 units, reflected across the \( x \)-axis, and shifted up 2 units. Write the resulting equation.

(A) \( y = -(x + 5)^3 + 2 \)  
(B) \( y = -(x^3 + 5) + 2 \)  
(C) \( y = -(x - 5)^3 + 2 \)  
(D) \( y = -(x - 5)^3 - 2 \)

74. The graph of the function \( y = 4(x + 2)^2 \) can be obtained from the graph of the function \( y = x^3 \) by which of the following transformations?

(A) Shift to the right by 2 units, then shift up by 4 units;  
(B) Shift to the left by 2 units, then stretch vertically by a factor of 4;  
(C) Shift to the right by 2 units, then stretch vertically by a factor of 4;  
(D) Shift to the left by 2 units, then stretch vertically by a factor of 4;  
(E) None of the above.
(25) Complex

75. Simplify the following expression in the standard form $a + bi$

(A) $(2 - 3i)^2$  
(B) $\frac{3-i}{1+2i}$

(26) The other

76. Which of the following statements is NOT correct about the polynomial $f(x) = -3x^5 + 4x^3 + x^2 + ax + 3$ (where $a$ is some real number)?

(A) It may have five local maximums and local minimums.
(B) It may have up to five zeros.
(C) Its graph has no asymptotes.
(D) 3 is its y-intercept.
(E) The right end of its graph goes down (to negative infinity).

77. Find the coordinate of the midpoint $M$ of $A(1,2)$ and $B(-6, -3)$

(A) $(-\frac{7}{2}, -\frac{1}{2})$  
(B) $(6, -1)$  
(C) $(-\frac{5}{2}, -\frac{1}{2})$  
(D) $(1,1)$

78. Rationalize the denominator : $\frac{5\sqrt{7}+1}{6}$

(A) $\frac{5\sqrt{7}}{6}$  
(B) $\frac{5\sqrt{7}-5}{6}$  
(C) $\frac{5\sqrt{7}-1}{6}$  
(D) $\frac{\sqrt{7}}{6}$

79. Find the quotient when $x^4 - 2x^2 + 3x + 4$ is divided by $x + 2$

(A) $x^3 - 2x^2 + 2x - 1$  
(B) $-2x^3 + 4x^2 + 10x + 23$  
(C) $x^2 - 4x + 11$  
(D) $-2x^2 + 2x - 1$

80. Find the remainder when $-2x^{40} + 5x^{20} + 4x^2 - 7x + 1$ is divided by $x + 1$ : Use the remainder theorem

(A) 15  
(B) 16  
(C) 1  
(D) 2

81. Which of the following is true?

(A) Its domain is $[-2,3]$
(B) Its range is $[-2,3]$
(C) It is only decreasing on the interval $(-2,3)$
(D) It is only increasing on the interval $(-2, -3)$
(E) The local max is $(3,5)$
**SOLUTION:**

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